Payments and cash management in the euro area: a quantitative analysis *

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Preliminary and incomplete

Abstract

Average cash holdings and the intensity of card usage widely differ across Euro Area regions. ECB payment diaries show that such gaps result from both supply-side differences (payments' acceptance by merchants) and demand-side ones (withdrawal and payment method choices). I present a dynamic cash management model featuring a payment method choice between cash and cards, uncertain lumpy expenditures/cash inflows, and imperfect acceptance of payment methods by merchants. I estimate the model at the region/province level by matching payment choice and cash management statistics, in order to quantify the drivers of geographical differences. I find that variation in card acceptance explains only around a third of geographical differences in card usage and it plays no role for the dispersion in cash holdings. The major drivers of heterogeneity in cash balances are differences in the cost of accessing and holding currency, while a large portion of gaps in card usage across regions are due to different perceived costs of using cards versus cash for point-of-sale payments.

Keywords: *cash management, payment choices, inventory model, card payments.*

JEL classifications numbers: *E41, E42, D14.*

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1 Introduction

Despite the extensive innovations in payment technologies that guaranteed access to free, secure and fast cashless means of payments to virtually all households, cash is still widespread in most economies: as documented by Shy (2023), as of 2019, ratios of currency in circulation to GDP range between 2% and 20% around the world, with many large economies showing as cash intensive, such as Japan (20%), the Euro Area (11%) and the United States (8%). While not all currency is held for transactional purposes[1] (exchanging goods and services), recent estimates (Esselink and Hernández (2017) for the Euro Area, Briglevics and Schuh (2021) for the US) reveal that households still extensively rely on cash to settle their purchases. That is not true for all countries, though: in Norway and Sweden, cash/GDP ratios fell close to 1%, consumers perform the vast majority of their transactions using cards, and many merchants are not willing to accept cash payments anymore. Cross-country differences in cash usage and means of payment choice have been documented since more than 30 years ago (Humphrey, Pulley, and Vesala (1996)), and they persist (in some cases, they have widened) until today. Even within geographical areas that shares the same currency, differences across countries are large, as shown by Bagnall et al. (2016) and Arango-Arango et al. (2018) comparing payment diaries for Austria, Germany, France, and the Netherlands. The scope of this paper is to shed light on such differences, using recent, detailed data from payment diaries for the Euro Area and a structural, quantitative approach.

In the first part of this paper, I provide new evidence of differences in cash holding and cash use across Euro Area regions, using transaction-level data coming from an harmonized daily payment diary rolled out by the European Central Bank in 2021-22. I start by documenting differences in cash holding patterns and payment behavior across 17 Euro Area countries, at the regional level (NUTS-1 or NUTS-2 level of aggregation). The extent of cross-regional variation along all these margin is significant: I show that, at the regional level of aggregation, average cash balances range from 75% to 185% of daily consumption, while the share of expenditure settled using cards ranges between 30% and 86%. The data allows me to elicit the micro-level determinants of regional differences by looking at the choices of individual households': I observe the frequency, size and

[1] Shy (2023) summarizes the reasons to hold cash above and beyond its use for transaction purposes, including hoarding cash as a store of value, or employing as a means to settle illicit activities (Rogoff (2017)) or to hide legal transactions in order to avoid paying taxes (Immordino and Russo (2018), Giammatteo, Iezzi, and Zizza (2022)).

timing of cash withdrawals, the amount of cash received from others (e.g. cash income), as well as payment method choices at points of sale between cash and cards. I also have data on merchant acceptance of cards and cash, as observe the set payment instruments accepted at the store for each transaction performed. I quantify and interpret geographical differences along all these margins. Some features of the data are unique and crucial: on one hand, the information on withdrawals enables me to fully characterize cash management policies followed by households (that cannot be inferred by data on average cash balances alone); on the other hand, the information on merchant acceptance enables me to disentangle differences in the intensity of card usage across regions due to supply-side constraints (cards being accepted more often in Finland than in Italy) from those due to consumer preferences (Finnish households using cards more often with respect to Italian ones when they have the chance to do so).

I then present a quantitative theory of cash management and payment choices, by laying out a structural inventory theoretical model designed to be taken to the data for estimation purposes. In the model, agents face a dynamic cost minimization problem: they have to finance an expenditure stream made of lumpy purchases whose frequency and size are stochastic, and they decide i) how much cash to hold at each point in time, paying a fixed cost for withdrawals and deposits and an opportunity cost on cash balances, and ii) how to pay for purchases, choosing between cash and cards. There are two state variables: the amount of cash held m and, in case of a transaction, the size of the payment faced s. While adjustment choices in between transactions only depend on m, payment method choices depend on both variables, as consumers compare the implied residual cash holdings in case of a cash payment ($m' = m - s$) with current cash balances m, and decide whether to pay cash or to use their payment cards and keep m on hand. We allow the fixed (per transaction) cost of using cash vs cards to be different, through an additive component κ that measures the cost/benefit of using cards with respect to cash. The model embeds several realistic frictions. First, not all merchants accept both cards and cash as means of payments: in some shops only cash can be used, while other stores are fully cashless and not willing to take cash anymore. Second, agents sometimes receive a random amount of cash without the need to withdraw it, resembling exogenous cash inflows such as income receipts in cash or currency received from family, friends, or in return for small amounts loaned. Third, agents decisions do not just depend on observed state variables, as I add random disturbances to the dynamic discrete choices faced by agents. This last modification yields two advantages: on one hand, it is convenient to establish the main identification result, linking model parameters to withdrawal and payment choice probabilities; on the other, it helps me to quantify how relevant are the state variables in determinining agents' choices relative to unmodeled features of the problem, by estimating the magnitude of shocks.

The last part of the paper is devoted to estimation. After externally calibrating some parameters that can be directly identified from the data under reasonable assumptions, I present an identification result for the set of structural parameters that must be estimated internally. I then estimate such models through a minimum distance procedure, matching relevant observable moments that summarize cash management and payment decisions of households. Parameters are estimated at the NUTS region level, in order to fully capture the degree of geographical differences displayed by the data. Overall, my estimates reveal substantial heterogeneity across Euro Area regions in the opportunity cost of accessing and holding cash, as well as in the relative cost of card payments with respect to cash ones. Opportunity costs are two to five times larger in France, Portugal and Spain than they are in most of Austria, Greece, Italy and Finland. In most countries, cash payments are still preferred to cash ones on average, with Estonia, Finland and parts of Belgium being the exceptions. The cost of accessing cash, measured by a model parameter that determines how costly it is to perform an unplanned cash withdrawal, is low in most of Greece, Italy, Latvia, Lithuania and part of Portugal, while substantially higher in several French, Finnish and Spanish regions. I show that most of the variation in the cost of accessing cash is within countries, while the bulk of differences in cash holding and cash usage cost are between countries.

I then use the model to perform a quantitative decomposition exercise, attributing to a number of factors (differences in expenditure streams, cash inflows, merchant acceptance probabilities, and structural parameters of the model) their contribution to the overall degree of observed cross-regional variation in average cash holdings and in the intensity of card usage. Results reveal that most differences in cash holdings across the Euro Area are due to variation between regions in the opportunity cost of holding cash and in the cost of accessing cash through withdrawals, while preferences for cards versus cash payments play a minor role. As for the intensity of card usage, the exercise reveals that only 30% of differences is determined by supply-side variation in the extent of merchant acceptance of cards and cash, while the remaining 70% of variation is mostly due to differences in the relative cost of paying with cards versus cash perceived by households. The bottom line of my quantitative exercise is that there are many sources of regional heterogeneity in cash holdings and payment behavior, and while some of them are attributable to market structure and supply-side forces (such as cards being accepted with varying degrees across different areas), most of the observed variation is due to characteristics of the environment in which people live (how costly is to obtain and hold cash given the available ATM infrastructure, interest rates and the probability of cash theft) and to the perceived relative costs/benefits of using cards versus cash to settle expenditures, in terms of fees, speed, privacy and security.

1.1 Related literature

This paper contributes to two strands of the literature on cash management and payment choices by households. The first is the mostly empirical literature that has investigated cross-country differences in payment choices and in the transactions demand for cash. Humphrey, Pulley, and Vesala (1996) compared payment patterns in fourteen developed economies using BIS data for the 19871-993 period, showing large differences in the mix of payment options used by consumers and attributing them (through fixed effects panel regressions) to a number of factors proxying both the availability of noncash transactions and institutional determinants, connecting the observed differences to varying degrees of violent crimes (which induce households to shy away from holding cash) and banking concentration (which makes electonic payments more accessible). More recently, Bagnall et al. (2016) rely on microdata from harmonized payment diaries covering seven countries (including four Euro Area countries, the US, Canada and Australia) to show large differences in cash holdings and cash usage by households, and they attribute part of such gaps to different rates of card acceptance across countries. I contribute to this literature in a twofold way: first, I use recent (2021-22), harmonized payment diaries covering a large portion of the Euro Area (all countries except Germany and the Netherlands), which provide new information on cash management and payment choice patterns of households and insights into the impact of imperfect merchant acceptance of payment methods on those choices; second, I complement the descriptive evidence on differences across areas with a structural estimation exercise which enables me to gauge the contribution of several factors in determining heterogeneity across regions and to perform counterfactual exercises.

The second strand of the literature I contribute to is that on theoretical models of cash management and payment choices. The seminal papers in this literature are due to Baumol (1952) and Tobin

(1956), whose model of cash management (Baumol-Tobin model, BT from now on) is been the benchmark framework that generated a vast literature on the so-called inventory-theoretical models of cash management. In the BT model, though, cash is the only available payment option for consumers. Others have augmented this framework to allow households to choose among cash and non-cash payments. Whitesell (1989) presents a static model of cash management with a non-degenerate size distribution of expenditures: agents need to decide which payments to pay in cash or using electronic payments, based on their size. Alvarez and Lippi (2017) present a dynamic inventory-theoretical model with an infinitesimal expenditure stream in which payment choices depend on the level of cash on hand. More recently, Briglevics and Schuh (2021) build a dynamic discrete choice model in which they allow agents to choose between cash, credit card and debit cards, depending on the size of the transaction and on the amount of cash on hand. Lippi and Moracci (2024) solve a continuous-time version of an inventory-theoretical model with withdrawal and payment method decisions, in which purchases arise at random times, have a non-degenerate size distribution and merchant acceptance of cards is imperfect. In this model, I build on the model by Lippi and Moracci (2024) augmenting it with features that enable me to take the model to the data. In particular, I add i) random disturbances to withdrawal and payment decisions, which generate adjustment probabilities à la Caballero and Engel (1993) instead of sharp sS rules for withdrawals, ii) exogenous cash inflows that occur randomly, à la Miller and Orr (1966), and iii) the possibility that while some merchants do not accept cards, other do not accept cash. The first feature is essential to gauge the contribution of unmodeled factors to the variation in observed choices and for identification purposes, while the second and the third additions are crucial to avoid attributing differences across countries to the wrong factors, as income received in cash and the imperfect acceptance of cash payments sharply vary between different areas.

1.2 Structure of the paper

The remainder of the paper is organized as follows. In Section 2 I introduce the data, present a series of motivating facts and then perform an empirical analysis that provides evidence of regional differences in cash management and payment choices across the Euro Area and explains their sources. In Section 3 I present the quantitative model, discuss the properties of optimal policies and derive model-implied moments. In Section 4 I bring the model to the data, first presenting the estimation strategy and the functional form assumption; then, I discuss the identification of the set of structural parameters and present the estimation results. In Section 5 I perform the decomposition exercise which is the main goal of the paper, highlighting the contribution of several factors in determining observed regional differences. Section 6 concludes.

2 Cash holdings and payment choices in the Euro Area

In this Section, I present the empirical analysis that provides the foundation for the quantitative exercise which is the core of the paper. In Section 2.1, I describe the data and present the main stylized facts that motivate my focus on geographical differences, documenting large regional heterogeneity in cash management and payment behavior across the Euro Area in 2021-22. In Section 2.2 I investigate the determinants of regional heterogeneity in average cash holdings. In Section 2.3 I empirically decompose differences in payment choices into their supply and demand-side components. Throughout the Section, I link my empirical findings to the inventory-theoretical literature on the transactions demand for cash.

2.1 Data and motivating facts

The data. I use data from the second wave of the Study on the Payment Attitudes of Consumers in the Euro Area (SPACE from now on), a survey rolled out in 2021-22 by the ECB to investigate the payment and cash management habits of Euro Area citizens. SPACE data provide a remarkable amount of information on cash management and payment decisions of a large number of households. I have data on more than 39,000 individuals coming from 17 different countries[2]. Respondents provide information about their purchases, payment choices and cash withdrawals in a given day, through a *payment diary*. Table 1 summarizes the most relevant variables which can be derived from the survey, aggregated at the region level[3]. For each variable, I report both its magnitude (through means and medians) and the extent to which it varies across regions (by display the coefficient of variation and the range). I start by displaying information on the daily expenditure flow e faced by agents, which averages around EUR 65. I then display average cash holdings M,

[2]I do not have access to data on two Euro Area countries, Germany and the Netherlands, as De Nederlandsche Bank and the Bundesbank run their own surveys and the related microdata are not publicly available.

[3]I observe a total of 100 different regions, which are defined at the NUTS-1, NUTS-2 or NUTS-3 level, depending on the country.

Table 1: Summary statistics at the region level.

	Mean	Median	SD	CV	Min	Max	N
N. respondents	376.83	340.50	240.99	0.64	42.00	1471.00	100
Expenditure stream							
Daily expenditure (EUR)	63.54	61.19	15.65	0.25	39.38	108.69	100
Daily cash received (EUR)	4.48	4.21	1.79	0.40	0.80	11.48	100
Cash holdings							
M (EUR)	73.20	71.01	14.99	0.20	44.23	112.59	100
M/e (days of expenditure)	1.19	1.16	0.24	0.21	0.76	1.85	100
Withdrawals, size and frequency							
W (EUR)	97.48	93.49	25.99	0.27	42.90	183.00	100
W/e (days of expenditure)	1.57	1.56	0.43	0.27	0.92	3.70	100
N. withdrawals per year	93.73	92.64	29.74	0.32	24.55	186.06	100
Cash at withdrawal							
$\underline{M}$ (EUR)	65.72	65.89	16.47	0.25	29.00	114.52	100
$\underline{M}/e$ (days of expenditure)	1.07	1.01	0.30	0.28	0.51	2.23	100
Available payment options							
Share accept only cash (merchants)	0.16	0.15	0.06	0.38	0.04	0.38	100
Share accept only cards (merchants)	0.06	0.06	0.04	0.62	0.00	0.18	100
Share accept both cash and cards (merchants)	0.77	0.79	0.07	0.09	0.57	0.88	100
Share own cards (consumers)	0.99	0.99	0.01	0.01	0.97	1.00	100
Preferences							
Share prefer cards (%)	0.59	0.61	0.08	0.14	0.35	0.81	100
Payment choices							
Share cards (% transactions)	0.43	0.40	0.12	0.28	0.21	0.80	100
Share cards | Both possible (% transactions)	0.39	0.38	0.11	0.27	0.22	0.71	100
Share cards (% of expenditure)	0.55	0.53	0.12	0.22	0.30	0.86	100
Share cards | Both possible (% of expenditure)	0.45	0.44	0.12	0.26	0.19	0.79	100

Note: The Table summarizes some of the key variables available in SPACE data. Information is aggregated at the region level, to remove variation within regions and better illustrate area disparities. The level of geographic aggregation is the NUTS-1/2/3 region level (100 regions in total). Regions are weighted according to their population (most recent estimates).
Source: Own calculations based on the Study on the Payment Attitudes of Consumers in the Euro Area (2021-22).

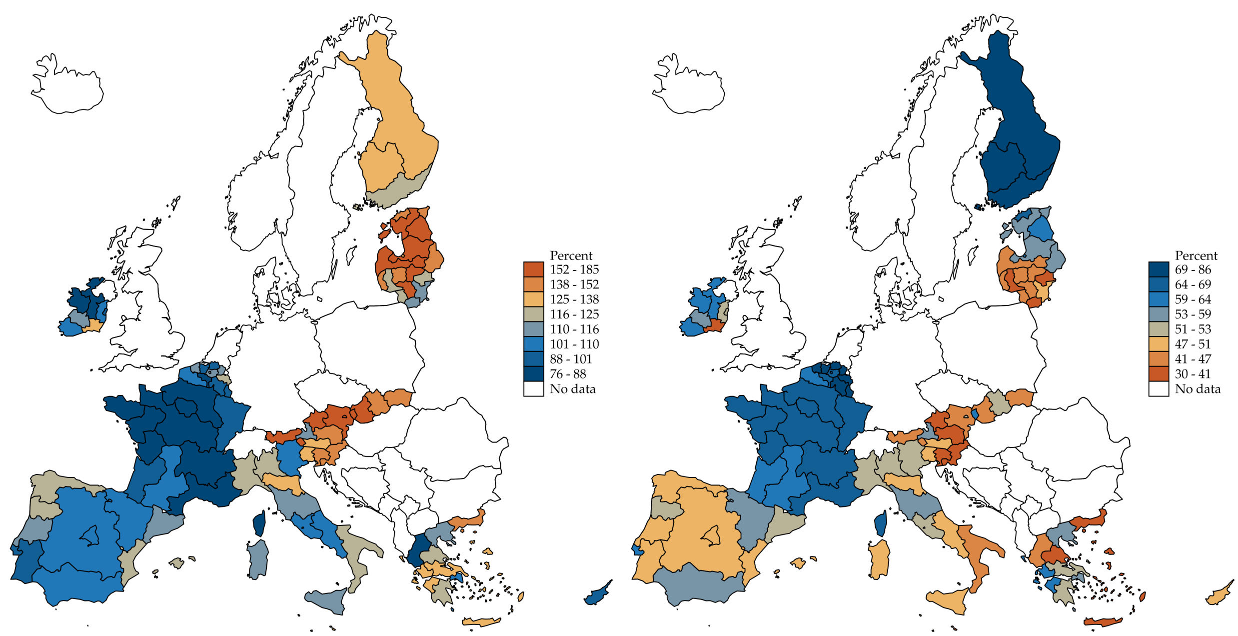
both in absolute terms and in terms of *days of expenditure* that they could be used to finance, i.e., relative to e. This normalization is used throughout the paper and makes sure that comparisons across regions are meaningful, as regional disparities could be simply due to differences in prices or on spending patterns, which I are not interested in. My goal is indeed to compare how different are cash management and payment choices by households across regions *in real terms*. Average cash balances are close to EUR 75, or around 120% of total daily expenditure. The data also provides information on the size and frequency of withdrawals. The mean size of a withdrawal W is around EUR 100, i.e., close to 160% of total daily expenditure, and households withdraw cash around 95 times a year on average (one withdrawal every four days, approximately). As it is possible to follow the path of cash balances throughout the diary day, one can identify the average amount of cash on hand when withdrawing $\underline{M}$, which is approximately EUR 65 (105% of daily expenditures). Importantly, survey respondents are asked about payment methods accepted in each of the points of sale in which they complete transactions during the diary day. This enables me to compute the shares of merchants that accept only cash, only cards, or both payment methods in their stores. In 2021-22, only in 77% of transactions both cash and cards were accepted at the store. A significant share of stores (16%) don't accept card payments, while about 6% are not willing to accept cash anymore. Access to cards by households, on the other hand, is near-universal, with 99% of households owning at least a cashless payment method. Respondents also answer a survey questionnaire where they are asked about their preferred payment method: 60% of individuals report that they prefer to use cards. I then display payment method shares, expressed as the percentage of transactions (expenditures) settled using cards. As I am able to observe the set of available payment options for each purchase, I also compute card shares for *unforced* transactions, i.e., transactions in which both payment methods were available to consumers (they had enough cash, and both cards and cash were accepted at the store). Households settle the majority of transactions using cash, with cards used only for 43% of purchases. When both options are available, this percentage falls even more, showing that a lot of card transactions arise as a result of cash balances being insufficient to carry out a purchases. When expressed as percentages of expenditure, card shares rise to 55% (45% for unforced expenditures), as individuals disproportionately carry out larger purchases using cards (Klee, 2008).

Regional differences: an overview. Figure 1 shows our main motivating facts. Panel A displays the average amount of cash balances divided by average daily expenditure in each region[4]. The plot shows that Europeans from different regions hold very dissimilar amounts of cash: normalized cash balances range from around 240% (Alyatus region, Lithuania) to 78% (Champagne-Ardenne region, France) of average daily expenditures. Panel B summarizes payment choice patterns across the Euro Area: for each region, I display the percentage of expenditures paid for using cash.

[4]I normalize cash balances using average daily expenditures at points of sale (i.e., excluding online purchases, recurring payments and peer-to-peer transactions) to obtain a comparable measure of cash balances that takes into account differences in the price level and in expenditure patterns. This metric quantifies the time (express in days) that the average household would take to deplete their average cash balances if they were to only use cash to settle expenditures. Notice that I could also have used the metric M/c, dividing cash balances by *cash* expenditure only. However, this would get rid of the portion of differences in M/e that arise as a result of heterogeneity in payment behavior, which are part of the variation I am interested in. I display the geographical distribution of M/c in Figure A.3b in the Appendix.

Figure 1: Cash management and payment choices across Euro Area regions.

(a) Average cash holdings (% of daily expenditure) (b) Card share of expenditures (%)



Note: In the left panel, I plot the quantity M/e (multiplied by 100), where M are average cash balances reported during the diary day by each respondent, and e is the average reported daily expenditure. In the right panel, I display the average share of expenditure settled using cards, which will be denoted by γ in the remainder of the paper. The level of geographic aggregation is the NUTS-1/2/3 region level (100 regions in total). In both panels, regions are split in eight quantiles and colored according to the quantile they belong to.
Source: Own calculations based on the Study on the Payment Attitudes of Consumers in the Euro Area (2021-22).

Again, warm colors denote a higher reliance on cash, i.e., a lower share of expenditure settled using cashless instruments. Payment method choices also differ substantially across regions: the share of expenditure settled using cards ranges between 0.19 (in the Helsinki-Uusimaa region of Finland) and 0.73 (in Malta). Several interesting patterns emerge by comparing the two panels. First, there seem not to be a necessarily increasing relationship between how much cash households hold on average (relative to their expenditure flow) and how intensely they use it as a payment device. While in some countries (such as France and Austria) the level of cash balances seem to be strongly associated with its usage as a payment method, this does not seem to be true for Finland, Italy and Spain, among others. Second, while regions in some countries (such as France and Spain) look relatively similar, cross-regional variation is prevalent in other countries, like Italy and Greece. In the Euro Area, near-cashless societies co-exist with cash intensive ones, sometimes within the same nation.

The focus on area differences. It is worth discussing why I focus on geographical differences across countries/regions instead of exploring other dimensions of heterogeneity across individuals. There are two main reasons that justify such a choice. First, there is already a large literature studying differences in cash balances and card usage across demographic groups defined in terms of education, income and age[5]. Second, I show through a decomposition exercise in Appendix A.1 that geography explains a larger share of variation in cash holdings and payment choices than demographics and reported preferences, suggesting that i) area differences are not simply the by-product of different demographic compositions across areas, and ii) individuals with similar demographic characteristics in different regions behave differently, plausibly as a result of differences in the payments environment, costs faced and available technologies.

2.2 Differences in cash holding patterns

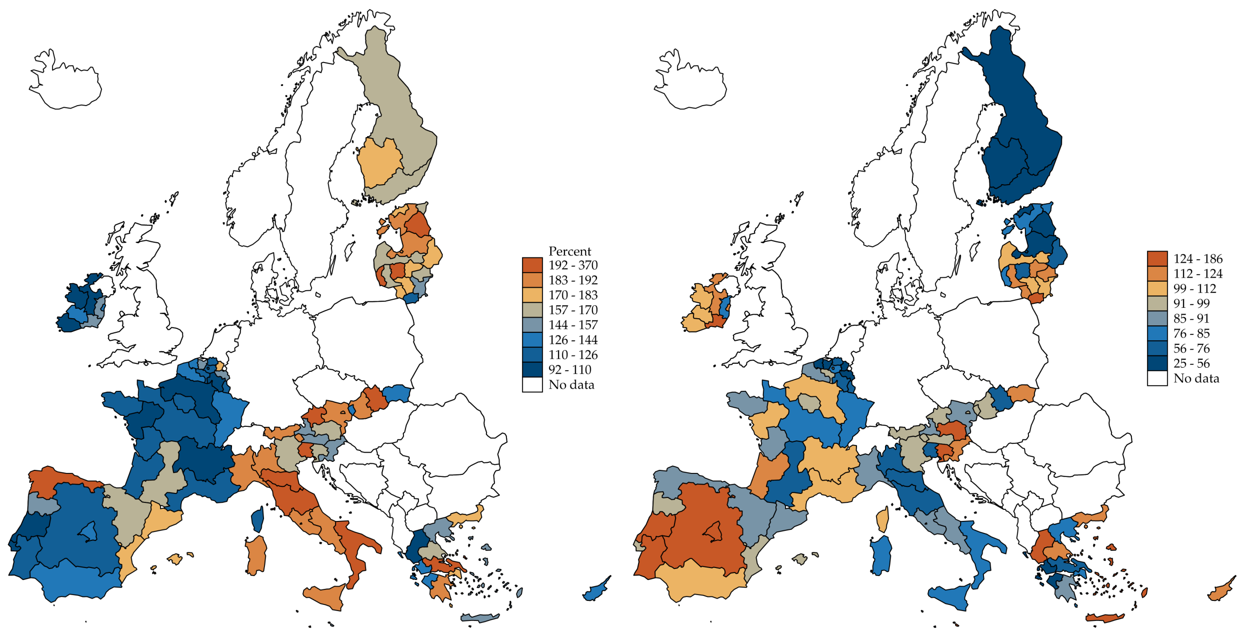
I now zoom into differences in average cash holdings by looking at cash management policies by households, which can be summarized through a set of summary statistics which are commonly used in the empirical and theoretical literature on the transactions demand for cash. A first set of statistics describe the properties of withdrawal choices by households: I provide evidence on the average withdrawal size W, on the number of withdrawals per year n, and on the average level of cash balances when withdrawals take place $\underline{M}$. These three objects are informative about three features of cash replenishments: they respectively tell us how much, how often and in which situations individuals decide to withdraw. The fourth and last statistic I provide describes how much cash is obtained by households without visiting ATMs, i.e., through cash inflows (e.g., cash income, or cash receipts from family, friends and colleagues). While the former three statistics reflect households' choices, the latter gives us information about features of the environment they live in (for instance, how frequent it is to receive part of their salary in cash in their area).

Cash withdrawals. I start by looking at *how much* do people withdraw across the Euro Area when they replenish their cash balances. In Figure 2a, I display the geographical distribution of the average withdrawal size W; as before, I normalize by daily expenditure and plot W/e. The Figure shows that the average withdrawal is worth between 2 and 4 days of expenditure in Italy, Southern

[5]See Stavins (2017) for a complete literature review that discusses the vast empirical work linking payment choices and cash demand to consumer demographics.

Figure 2: Differences in cash holding patterns - I

(a) Average withdrawal (% of daily expenditure) (b) Number of withdrawals per year



Note: In the left panel, I plot the quantity W/e (multiplied by 100), where W is the average withdrawal size reported during the diary day by respondents that do withdraw, and e is the average reported daily expenditure. In the right panel, I display the average number of withdrawals per year n. The level of geographic aggregation is the NUTS-1/2/3 region level (100 regions in total). In both panels, regions are split in eight quantiles and colored according to the quantile they belong to.
Source: Own calculations based on the Study on the Payment Attitudes of Consumers in the Euro Area (2021-22).

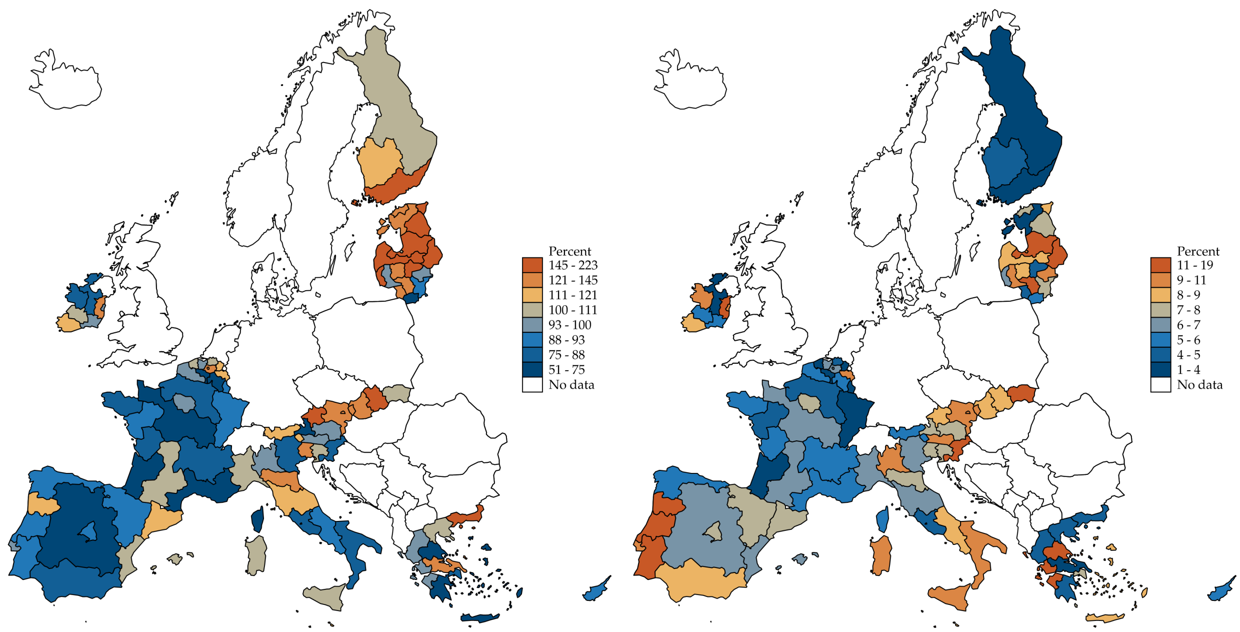
Greece and the majority of Austria, Slovakia and the Baltics, while it is enough to cover less than 1.5 days of expenditure (in some case even less than one day), in most of France, Portugal, Spain and Ireland. In Figure A.4a in the Appendix I display the geographical distribution of the ratio W/M, an alternative measure of withdrawal sizes which is commonly presented in the literature on cash demand. Across the Euro Area, W/M ranges between 0.8 and 2.2, with most regions display a value between 1 and 2[6]. These are smaller than the ratio W/M implied by the conventional BT model (which is equal to 2), but consistent with inventory models of money demand that feature a precautionary motive for holding cash, such as Alvarez and Lippi (2009) or Alvarez and Lippi (2013).

In Figure 2b, I plot the average number of withdrawals per year n across Euro Area regions. Again, the choices of households from different areas look very different quantitatively. The yearly number of trips to the ATM ranges between 24 (twice per month, in the Helsinki area)

[6]The estimated levels of W/M for Austria and France are comparable in terms of magnitudes with estimates by Bagnall et al. (2016) using 7-day payment diaries for 2011, which is reassuring regarding the quality of the information I elicit through SPACE.

Figure 3: Differences in cash management policies.

(a) Cash at withdrawal (% of daily expenditure) (b) Daily cash inflow (% of daily expenditure)



Note: In the left panel, I plot the quantity $\underline{M}/e$ (multiplied by 100), where $\underline{M}$ is the average level of cash balances at withdrawals, and e is the average reported daily expenditure. In the right panel, I display the average normalized daily cash inflow i/e (multiplied by 100). The level of geographic aggregation is the NUTS-1/2/3 region level (100 regions in total). In both panels, regions are split in eight quantiles and colored according to the quantile they belong to.
Source: Own calculations based on the Study on the Payment Attitudes of Consumers in the Euro Area (2021-22).

and 186 (every two days, in the Ipeiros region of Greece). Cash withdrawals are particularly infrequent in Finland, Estonia and Belgium, while the opposite holds for Portugal, Spain and other Mediterranean countries. As one can easily spot by comparing the two maps, the normalized size W/e and frequency of withdrawals n are clearly negatively correlated. Indeed, in the BT model of cash management, as well as in extensions such as Alvarez and Lippi (2009), they are perfectly invesely related through the identity $nW = 365e$. The correlation in the above maps looks far from perfect, though. As I discuss in Appendix A.2 revisiting the BT accounting identity to allow for card payments and exogenous cash inflows, the negative correlation between n and W/e is imperfect because of regional differences in the amount of cash received from others per year r, as well as variation in γ, the share of expenditures settled using cards.

Lastly, in Figure 3a, I plot the average level of cash balances when a withdrawal takes place (normalized by daily expenditure) $\underline{M}/e$. The graph shows that in some regions people visit ATMs in order to replenish their cash holdings when they barely hold enough cash to finance less than a day of expenditures. In most of Portuguese, French and Greek regions, this is the average household behavior. In other areas, such as Northern Italy, Austria, Slovakia, Baltic countres and Finland, people withdraw when they much higher amounts of cash on hand, enough to finance more than one and in some cases even two days of expenditure. In Figure A.4b in the Appendix I display the geographical distribution of the ratio $\underline{M}/M$, which ranges between .42 and 1.38. As for W/M, these figures are inconsistent with the deterministic BT model, in which $\underline{M} = 0$, but broadly compatible with models of cash management which feature a precautionary motive for holding cash and therefore allow for cash withdrawals when individuals still have cash on hand. It is worth noticing, though, that for some regions I observe values of $\underline{M}/M$ larger than one[7], which are also inconsistent with Alvarez and Lippi (2009), who show that $\underline{M}/M \leq 1$ in their BT model augmented with free withdrawal opportunities.

Cash inflows. As a last determinant of average cash balances, I focus on the role played by exogenous cash inflows. In SPACE data, several types of such inflows are reported, including income received in physical cash, pocket money received from parents, peer-to-peer payments between friends, family members and colleagues. In Figure 3b I report the ratio of daily cash inflows to total daily expenditure across Euro Area regions. The map shows that there is a large degree of heterogeneity in the intensity of cash inflows between different countries, and even within countries. Daily cash inflows range between 1 and 20 percent of daily expenditures, with Latvia and Portugal ranking among the countries where households receive the most cash inflows, and Finland and France on the opposite side of the spectrum. Within some countries, regions are very different: this happens in Italy, where cash inflows are way more common in the South than in the North, but also in Greece and Ireland.

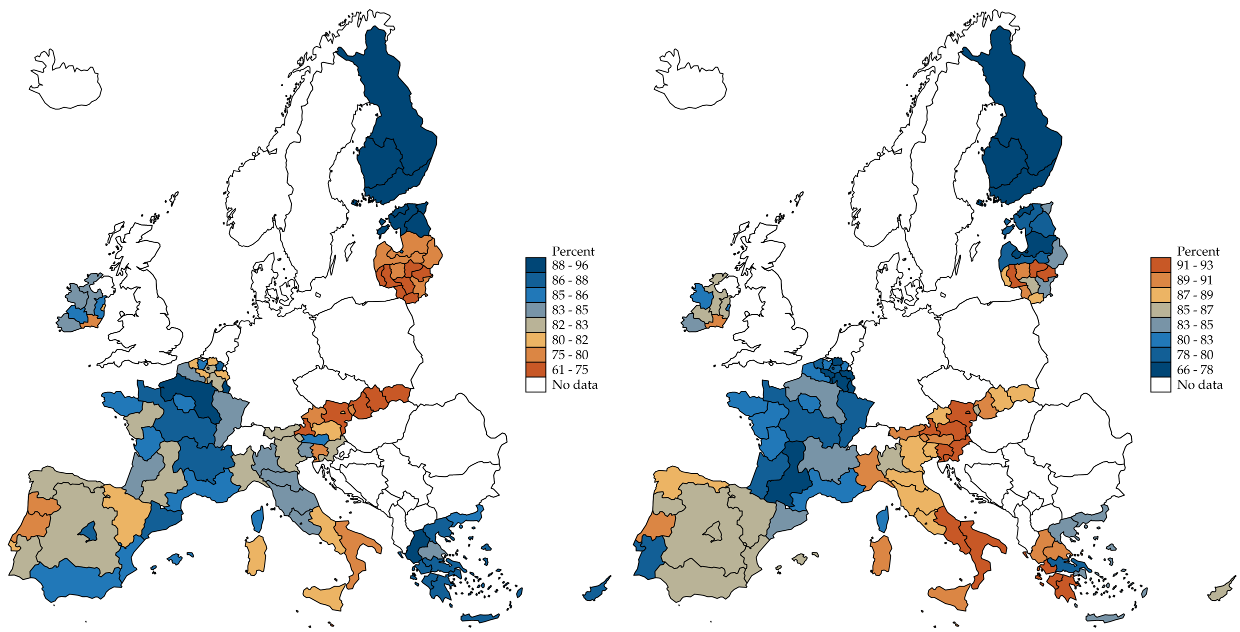
2.3 Differences in payment behavior

I now turn to examining differences in payment behavior at the point of sale, summarized through the share of expenditures settled using cards. In Figure 1b, I already displayed its distribution across EA regions. In this Subsection, I try to analyze its determinants, by exploiting the rich information provided by SPACE. In particular, I disentangle differences in the intensity of card usage due to supply and demand-side choice constraints (imperfect merchant acceptance, non-universal access

[7]Values of $\underline{M}/M$ larger than one have been previously found by Bagnall et al. (2016) for the US and Netherlands. For Austria, moreover, I get values around .75-.85, which are perfectly in line with 2011 estimates reported by Bagnall et al. (2016).

Figure 4: Differences in payment choice constraints.

(a) Card payment possible (% purchases) (b) Cash payment possible (% purchases)



Note: In the left panel, I plot the share of reported purchases in SPACE in which using a payment card was an available payment options for the respondent (the household has access to a cashless device and the merchant accepts card payments). In the left panel, I plot the share of reported purchases in SPACE in which using cash was an available payment options for the respondent (the household has enough cash on hand to complete the transaction and the merchant accepts cash payments). The level of geographic aggregation is the NUTS-1/2/3 region level (100 regions in total). In both panels, regions are split in eight quantiles and colored according to the quantile they belong to.
Source: Own calculations based on the Study on the Payment Attitudes of Consumers in the Euro Area (2021-22).

to card payments, and insufficient cash holdings) from variation in *unforced* payment choices, by looking at the intensity of card usage when both cash and cards were available options. I then exploit answers to survey questionnaires to link the intensity of card usage for unforced transactions to reported preferences for cash vs cards.

The role of choice constraints. For some transactions, using cards is not a feasible option for the buyer. This might happen because of demand or supply-side constraints. On the demand side, notice that in order to perform a card transaction, households must own either a debit or a credit card, or they must have access to an alternative cashless technology such as mobile payments. As I showed in Table 1, however, this is not an issue in the EA for the period 2021-22: almost all households own a payment card[8]. A second requirement is that cards have to be accepted at the point of sale in order to use them. Card acceptance rates display sizeable variation across regions in the EA, as I show in Figure A.5b in the Appendix. While acceptance is near-universal in areas

[8]The exact distribution of card ownership across regions is displayed in Figure A.5a in the Appendix.

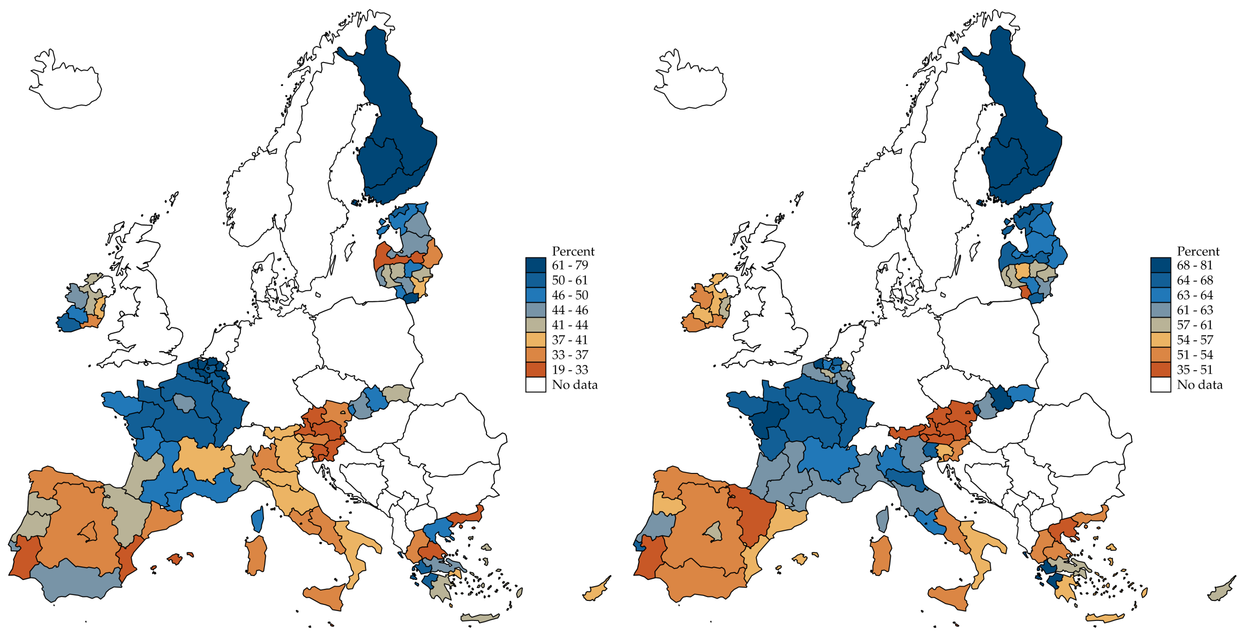
such as Finland and Estonia and quite widespread in France, Greece and Ireland, it is much less common in other regions, with Austria, Slovakia, Latvia and Lithuania being the countries that record the lowest share of shops that accept card payments. In Figure 4a I display, for each region, the share of purchases in which both the above requirements are satisfied, and therefore using a payment card is a feasible option for buyers. This occurs with particularly high frequency (for 85 to 95 percent of purchases) in Estonia, Finland, and parts of France and Greece. Using cards is instead often not an option in Latvia, Lithuania, Slovakia and Austria, as well as in parts of Italy, Portugal and Spain.

At the same time, cashless methods are always used as a means of payment whenever using physical currency is not possible. For cash to be an option, two conditions must hold: first, the shop must accept card payments; second, the amount of cash on hand should be larger than the size of the transaction. In Figure A.5c and Figure A.5d in the Appendix I display the share of transactions in which, respectively, the first and second condition are satisfied. In Figure 4b, I combine both conditions and I look at the share of purchases that could have been settled using cash by buyers, across regions. The Figure shows that a large portion of cashless payments arise because buyers cannot use cash. This happens quite often in Belgium, Estonia, Finland and France, as a result both of imperfect cash acceptance (especially in Belgium, Estonia and Finland, where approximately more than one shop out of ten doesn't accept cash anymore) and insufficient cash balances.

Unforced payment choices. In order to understand if households' payment behavior at POS is heterogeneous across the Euro Area, it makes the most sense to focus on *unforced* transactions, i.e., transactions in which both payment methods were available to the buyer. This happens under four conditions: i) the agent has sufficient cash to carry out the purchase, ii) the merchant accepts cash, iii) the agent has access to a payment card, and iv) cards are accepted by the merchant. For each region, I compute the average flow of expenditure that could have been paid using either cash or cards, and then I display the portion of this expenditure which ends up being paid using cards. I display the distribution of unforced payment choices (in value terms) in Figure 5a. The map makes it clear that even focusing on unforced transactions, the intensity of cashless usage sharply varies across countries. Table 1 actually shows that the coefficient of variation of unconstrained payment choices across regions is higher than that of overall payments, suggesting that choice constraints actually dampen regional heterogeneity. Conditional on having both payment options

Figure 5: Differences in unforced payment choices and reported preferences.

(a) Card share of unforced expenditures (%) (b) Card preferred to cash (% households)



Note: In the left panel, I plot the share of expenditure settled using a payment card for *unforced* purchases, i.e., purchases in which both payment options were available to the buyer. In the left panel, I plot the share of SPACE respondents that reportedly prefer to use cards at points of sale to settle their transactions when answering the question "If you were offered various payment methods in a shop, what would be your preference?". The level of geographic aggregation is the NUTS-1/2/3 region level (100 regions in total). In both panels, regions are split in eight quantiles and colored according to the quantile they belong to.
Source: Own calculations based on the Study on the Payment Attitudes of Consumers in the Euro Area (2021-22).

available, agents use card with to settle 85 to 95% of expenditures in most of Estonia, Finland, France and Greece, and in parts of Spain. In Northern Italy, mainland Spain and Ireland, 80 to 85% of expenditure are settled using cards. Finally, in parts of Italy and Portugal, and especially in Austria, Slovakia, Lithuania and Latvia, the percentage falls below 80% in in some regions is is as low as 60%. In Figure 5b I show that the choices we just discuss correlate with stated preferences in preferred payment methods, elicited through a survey question in SPACE. For each region, we display the percentage of households that reportedly prefer to use cards to settle purchases, which ranges from around 35% (in Austria and parts of Greece, Portugal and Spain) to approximately 80% (in most of Finland). Overall, unforced payment choices and reported preferences seem closely, suggesting that when having both options available individuals choose according to the costs and benefits they face when paying in cash versus cards.

3 A quantitative model

In this Section, I present a parsimonious quantitative theory of cash management and payment choices with the goal of building a framework that can be brought to the data in order to quantify the sources of regional differences in cash holdings and card usage across the Euro Area. In Section 3.1 I introduce the model setup, describing the sequence problem and the dynamic program faced by households. In Section 3.2 I define and illustrate the properties of optimal adjustment and payment method policies. Finally, in Section 3.3, I show how to obtain a set of moments implied by the solution of the model, which will be useful to estimate the model.

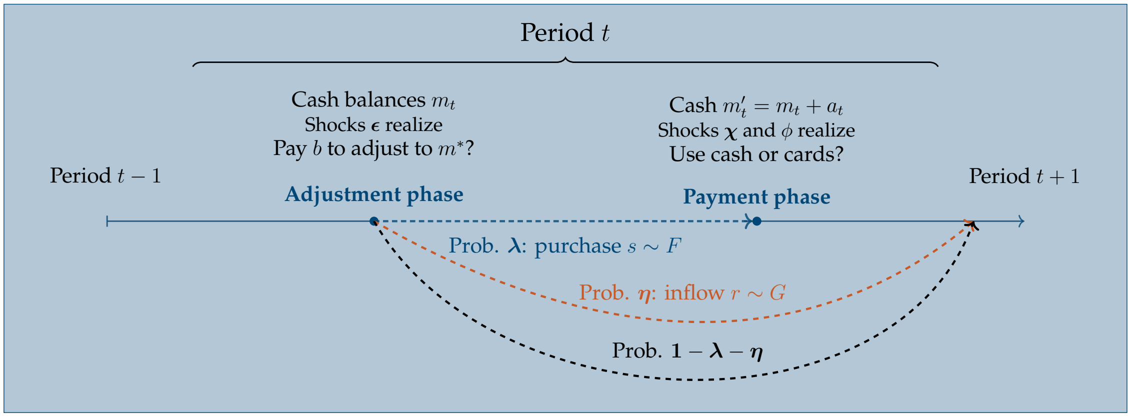
3.1 Model setup

Time is discrete and indexed by t. In each period t there are (potentially) two sub-periods, called *adjustment phase* and *payment phase*. Let m_t denote the level of cash balances in the adjustment phase, with $0 \leq m \leq \overline{m}$, with $\overline{m}$ being some finite maximum storage capacity. During the adjustment phase, agents decide whether to adjust (a) their cash balances (either withdrawing or depositing) by paying a fixed cost b, or to not adjust (n), after drawing additive taste shocks $\boldsymbol{\epsilon} = (\epsilon_a, \epsilon_n)$ for both options. After the adjustment phase is over, the payment phase starts. Let m'_t denote cash balances at the beginning of the payment phase. During the payment phase, with probability $\eta > 0$, agents receive an amount of cash r whose value is drawn from a continuous probability distribution with cdf $G(r)$ and support $(0, +\infty)$. With probability $\lambda > 0$, instead, agents need to purchase in the same period a good whose value s is drawn at random from a continuous probability distribution with cdf $F(s)$ and support $(0, +\infty)$. Finally, with probability $1 - \lambda - \eta$, nothing happens during the payment phase and the next adjustment phase begins.

When agents need to purchase something, they can settle the transaction either with cash (c) or using a cashless payment method (d), e.g., a debit card, or credit card, or mobile payment technology, whose usage entails a fixed cost κ with respect to cash usage[9]. A proportion ϕ_c of shops only accept cash payments, while a fraction ϕ_d are only willing to receive card payments; the remaining share of shops $\phi_{cd} = 1 - \phi_c - \phi_d$ accepts both payment methods. The size of the purchase, as well as the type of shop $\ell \in \{c, d, cd\}$ in which the purchase has to take place are

[9]In the estimation, I allow κ to be negative, i.e., for card payments to be perceived as cheaper than cash ones.

FIGURE 6: Timing of the model.



revealed at the same time at the start of the payment phase. Clearly, cash cannot be employed when the size of the purchase s is larger than current cash balances m. When both payment options are available, agents draw additive taste shocks $\boldsymbol{\chi} = (\chi_a, \chi_n)$ for both options and then decide which payment method to use. Finally, if agents face a purchase of size larger than m when visiting a shop of type c, they need to immediately visit an ATM and withdraw cash to finance the purchase, paying a fixed cost ψb, where $\psi > 1$ is a penalty for such *unplanned* withdrawals.

Sequence problem. I now describe the problem formally. Let m denote the level of cash balances. Holding m units of cash entails an opportunity cost Rm, while adjusting cash holdings through a withdrawal has a fixed cost $b > 0$. The future is discounted at rate $\beta < 1$. Agents face the sequence problem

$$\begin{aligned}
\max_{\{w_{\tau_i}, \tau_i, \widehat{\tau}_i, \widetilde{\tau}_i\}_{i=0}^{\infty}} \quad & \mathbb{E}\left\{ \sum_{t=0}^{+\infty} \beta^t \left(Rm_t' + \varepsilon_t + \mathbb{1}\left(s_t > 0\right) \chi_t \right) + \sum_{i=0}^{\infty} \left(\beta^{\tau_i} b + \beta^{\widehat{\tau}_i} \kappa + \beta^{\widetilde{\tau}_i} \psi b \right) \,\middle|\, m_0 = m \right\} \\
\text{subject to} \quad & m_t = m_0 + \textstyle\sum_{\tau \le t} \left(a_\tau + a_\tau'\right) - \sum_{\tau \le t} s_t \mathbb{1}\left(p_t = 0\right) + \sum_{\tau \le t} r_t, \\
& m_t' = m_t + a_t,
\end{aligned} \tag{1}$$

where m_t (m_t') is the level of cash balances in the adjustment (payment) phase of period t, R is the opportunity cost of holding cash, b is the cost of performing a withdrawal/deposit, κ is the cost of using cards, ψ is the penalty for unplanned withdrawals, a_t (a_t') is the amount withdrawn (deposited if negative) in the adjustment phase (payment phase) of period t, s_t is the size of the purchase faced in period t ($s_t = 0$ meaning that there was no need to purchase anything in that period), r_t is the amount of cash received in period t, and $p_t = 0$ indicates that the purchase opportunity of period t has been paid for using cash. I denote by $\{\tau_i, \widehat{\tau}_i, \widetilde{\tau}_i\}_{i\in\mathbb{N}}$, respectively, the *stopping times* at which planned withdrawals occur (τ_i), card payments are carried out ($\widehat{\tau}_i$), or unplanned withdrawals happen ($\widetilde{\tau}_i$). Finally, I denote by ε_t the utility derived by the taste shock associated with the chosen adjustment alternative, and χ_t has a similar meaning for taste shocks related to payment choices. The constraints in (1) highlight how the law of motion of m is affected both by the exogenous occurrence of cash inflows and by agents' choice on when and how much to withdraw and on how to pay.

Dynamic programming problem. At the beginning of the adjustment phase, an agent with cash holdings m needs to choose an action $j \in \{a, n\}$ (adjust - not adjust). Their problem at this stage is given by

$$V(m, \boldsymbol{\epsilon}) = \max\left\{v_a + \epsilon_a\ ,\ v_n(m) + \epsilon_n\right\}, \tag{2}$$

where $\boldsymbol{\epsilon} = (\epsilon_a, \epsilon_n)$ is a vector of mean-zero, Gumbel-distributed disturbances with scale parameter σ_b, which are meant to capture determinants of adjustment choices other than the current level of cash balances[10]. The distributional assumption on (ϵ_a, ϵ_n) yields an analytical expression for the expected value of V, given by

$$EV(m) = \mathbb{E}_{\boldsymbol{\epsilon}}\left(V(m, \boldsymbol{\epsilon})\right) = \log\left(\exp\left(\frac{v_a}{\sigma_b}\right) + \exp\left(\frac{v_n(m)}{\sigma_b}\right)\right)\sigma_b. \tag{3}$$

The choice-specific utility associated with inaction ($j = n$) is given by

$$\begin{aligned} v_n(m) = &- Rm + \lambda U(m) + \\ &+ \eta\left(\beta\int_0^{\overline{m}-m} EV(m+r)g(r)ds + (1 - G(\overline{m} - m))\left(\beta\max_{\widehat{m}} EV(\widehat{m}) - b\right)\right) \\ &+ (1 - \lambda - \eta)\beta EV(m), \end{aligned} \tag{4}$$

where $U(m)$ is the expected utility associated with facing a payment when cash balances are m, defined later. Notice that when the cash inflow is so large that future cash balances would exceed the maximum storage capacity $\overline{m}$, a forced deposit takes place and agents reset their cash balances to any desired amount m'. The choice-specific utility associated with adjustments ($j = a$) is instead

[10] I assume $\epsilon_j \sim \text{Gumbel}(-\gamma\sigma_b, \sigma_b)$, where $\gamma \simeq 0.5772$ is the Euler-Mascheroni constant. Recall that if $e \sim \text{Gumbel}(\mu, \sigma)$, then $\mathbb{E}(e) = \mu + \gamma\sigma$ and $\text{SDev}(e) = \frac{\sigma\pi}{\sqrt{6}}$. Given our rescaling, I have that $\mathbb{E}(\epsilon) = 0$

given by

$$v_a = \max_{\widehat{m}} v_n(\widehat{m}) - b, \tag{5}$$

where b is the fixed cost of adjustment. Notice that v_a is independent from m, due to the absence of variable adjustment costs. Let $m^* = \arg\max_{\widehat{m}} v_n(\widehat{m}) = \arg\max_{\widehat{m}} EV(\widehat{m})$ denote ex-ante optimal cash holdings[11]. Notice that after both planned and unplanned adjustments of cash agents will always hold the same amount of cash m^*.

In the payment phase, it might be that the agent has both payment options available, and that she needs to choose an action $k \in \{c, d\}$ (pay cash - pay using card). This happens when $s \leq m$ and the store visited is of type $\ell = cd$. In this situation, the value of the problem for an agent with cash balances m facing a purchase of size s when having both options available is given by

$$W(m, s, \boldsymbol{\chi}) = \max\{v_c(m, s) + \chi^c, v_d(s) + \chi^d\}, \tag{6}$$

where $\boldsymbol{\chi} = (\chi^c, \chi^d)$ is a vector of mean-zero, Gumbel-distributed disturbances with scale parameter σ_κ, which are meant to capture determinants of payment choices other the amount of cash holdings and the size of the transaction. Given this distributional assumption on (χ^c, χ^d), the expected value of $W(m, s)$ is given by

$$EW(m, s) = \mathbb{E}_{\boldsymbol{\chi}} W(m, s, \boldsymbol{\chi}) = \log\left(\exp\left(\frac{v_c(m, s)}{\sigma_\kappa}\right) + \exp\left(\frac{v_d(m)}{\sigma_\kappa}\right)\right)\sigma_\kappa, \tag{7}$$

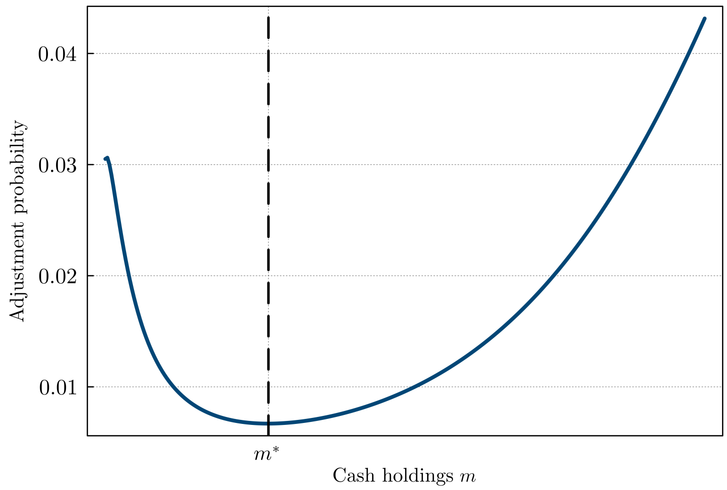
and the choice-specific utility associated to each of the choices are given by

$$\begin{aligned} v_c(m, s) &= \beta EV(m - s), \\ v_d(m) &= \beta EV(m) - \kappa, \end{aligned} \tag{8}$$

where the choice-specific utility associated with card payments is independent of the size of the purchase, as it does not impact future cash balances. Therefore, the expected value of the problem

[11]Notice that EV and v_n have the same global maxima as $EV(m)$ is an increasing transformation of $v_n(m)$.

FIGURE 7: Adjustment probabilities $\pi_a(m)$.



at the beginning of the payment phase when cash balances are equal to m is given by

$$
\begin{aligned}
U(m) = \; & \phi_d v_d(m) + \phi_c \left[(1 - F(m)) \left(\beta \max_{\widehat{m}} EV(\widehat{m}) - \psi b \right) + \int_0^m v_c(m,s) f(s) ds \right] + \\
& + \phi_{cd} \left[(1 - F(m)) \, v_d(m) + \int_0^m EW(m,s) f(s) ds \right].
\end{aligned} \tag{9}
$$

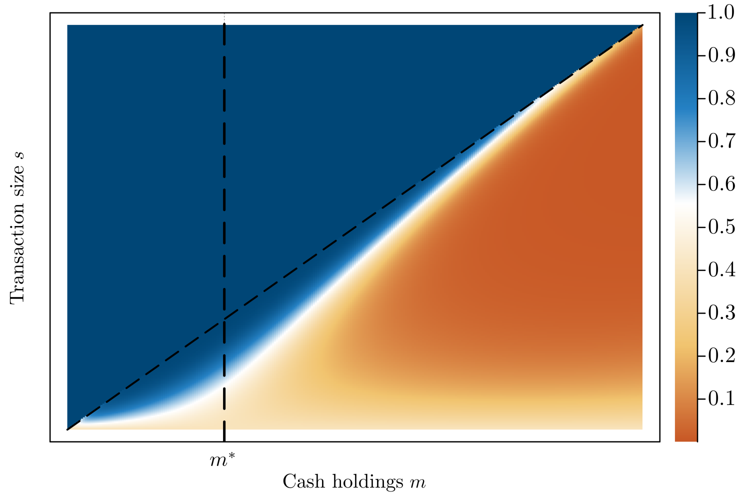
When the shop visited does not accept cards, and cash is not enough to complete the transaction, agents are forced to withdraw at least the amount of cash needed to carry out the purchase. In such situations, when facing a purchase $s > m$, they will withdraw $m^* + s - m$, in order to start the next period with the optimal amount of cash after having completed the transaction. As said before, such *unplanned* withdrawals are more costly that normal ones, with $\psi > 1$ being the extra cost of unplanned withdrawals.

3.2 Optimal decision rules

I now discuss the features of optimal decision rules in this setting. There are three sets of decisions rules, namely i) the optimal return value m^*, ii) adjustment choice probabilities and iii) payment choice probabilities.

Adjustment policy. I start by discussing the properties of optimal adjustments of cash balances. Notice that given the presence of unobserved taste shocks, policy functions are expressed as choice probabilities rather than as deterministic objects. Under our distributional assumptions on the

FIGURE 8: Card usage probabilities $\pi_d(m, s)$.



unobserved determinants of adjustment decisions, *adjustment probabilities* are given by

$$\pi_a(m) = \frac{\exp\left(\frac{v_a}{\sigma_b}\right)}{\exp\left(\frac{v_a}{\sigma_b}\right) + \exp\left(\frac{v_n(m)}{\sigma_b}\right)} = \frac{\exp\left(\frac{v_n(m^*)-b}{\sigma_b}\right)}{\exp\left(\frac{v_n(m^*)-b}{\sigma_b}\right) + \exp\left(\frac{v_n(m)}{\sigma_b}\right)}. \tag{10}$$

Notice that, as $\sigma_b \to +\infty$, I have that $\pi_a(m) \to 1/2$, as cash management becomes an irrelevant determinant of decisions: only taste shocks matter. Instead, as $\sigma_b \to 0$, I have that $\pi_a(m) \to 1$ when $v_n(m) < v_n(m^*) - b$, and $\pi_a(m) \to 0$ when the opposite inequality holds. When agents decide to adjust their cash balances, they always reset them to m^*. The size of withdrawals (when different from zero) is therefore given by by $w(m) = m^* - m$, and the size of deposits is $d(m) = m - m^*$. Notice that differently from the majority of models of cash management[12], in this framework deposits can optimally occur due to the presence of exogenous cash inflows that may push cash balances above their optimal level. I display the shape of $\pi_a(m)$ for an intermediate value of σ_b in Figure 7. These U-shaped adjustment probabilities that rise as m departs from m^* are reminiscent of generalized hazard functions à la Caballero and Engel (1993): indeed, the presence of random disturbances makes the problem equivalent to one with random variation in the fixed cost b, or other mechanisms that generate adjustment hazards instead of sharp (s, S) rules, in which agents instead adjust with probability one only when the state crosses some fixed lower or upper boundary.

[12]In the standard cash management model à la Baumol (1952) and Tobin (1956), there are no exogenous inflows of cash. Examples of models with random cash receipts are Miller and Orr (1966) and Eppen and Fama (1969), among others.

Payment choice policy. When cash balances in the payment phase are larger than the size of the incoming purchase, agents face a nontrivial payment method choice. Under the distributional assumptions on payment taste shocks χ, *card usage probabilities* are given by

$$\pi_d(m,s) = \frac{\exp\left(\frac{\beta EV(m)-\kappa}{\sigma_\kappa}\right)}{\exp\left(\frac{\beta EV(m)-\kappa}{\sigma_\kappa}\right) + \exp\left(\frac{\beta EV(m-s)}{\sigma_\kappa}\right)}. \tag{11}$$

Again, notice that when $\sigma_\kappa \to +\infty$, $\pi_d(m,s)$ approaches 1/2, as before. The heatplot in Figure 8 displays card usage probabilities $\pi_d(m,s)$. Notice that agents rely more frequently on cash when the size of the transaction is small relative to m, while for large transactions cards are preferred. When the amount of cash held is large relative to optimal cash balances m^*, agents use cash for almost all purchase sizes as they want to get rid of it.

3.3 Model-implied moments

After computing the optimal decision rules $\{m^*, \pi_a(m), \pi_d(m,s)\}$, it is possible to obtain a set of statistics summarizing the cash management and payment choices of a cross-section of agents, without the need for simulation. The first key object that needs to be computed in order to perform such task is the stationary distribution of cash holdings. This can be seen both as the long-run distribution of cash holdings of a single agent, and as the cross-sectional distribution of cash balances at each point in time. Notice that since in the model each period has two sub-periods, there are going to be *two* stationary distributions of cash balances, one for the adjustment phase ($h^a(m)$) and one for the payment phase ($h^p(m)$). As in Lippi and Moracci (2024), the two stationary distributions have a mass point at m^*, which is going to be pinned down by the boundary conditions

$$\lim_{\varepsilon\to 0_+} \left(\int_0^{m^*-\varepsilon} h_t^a(m) + \int_{m^*+\varepsilon}^{\overline{m}} h_t^a(m)\right) + h_t^a(m^*) = 1, \tag{12}$$

$$\lim_{\varepsilon\to 0_+} \left(\int_0^{m^*-\varepsilon} h_t^p(m) + \int_{m^*+\varepsilon}^{\overline{m}} h_t^p(m)\right) + h_t^p(m^*) = 1, \tag{13}$$

where the mass points are denoted by $h_t^a(m^*)$ and $h_t^p(m^*)$, with a slight abuse of notation, and they are probabilities. From now on, to ease the notation, I write the above two limits simply as $\int_0^{m^*} mh^k(m)dm$ and $\int_{m^*}^{\overline{m}} mh^k(m)dm$, respectively, keeping in mind that the point m^* is always excluded when I compute integrals. The two distributions can be obtained as the fixed point of a system of difference equations[13]. For $m \neq m^*$, I have

$$
\begin{aligned}
h_t^a(m) = \quad & (1-\lambda-\eta) h_{t-1}^p(m) + \lambda \phi_d h_{t-1}^p(m) + \\
& \lambda \phi_c \left(\int_m^{+\infty} h_{t-1}^p(\widehat{m}) f(\widehat{m}-m) \mathbb{1}\left(\widehat{m} \neq m^*\right) d\widehat{m} + h_{t-1}^p(m^*) f(m^*-m) \right) + \\
& \lambda \phi_{cd} h_{t-1}^p(m) \left(1 - F(m) + \int_0^m \pi_d(m,s) f(s) ds \right) + \\
& \lambda \phi_{cd} \Bigg(\int_m^{+\infty} h_{t-1}^p(\widehat{m}) f(\widehat{m}-m) \left(1-\pi_d(\widehat{m}, \widehat{m}-m)\right) \mathbb{1}\left(\widehat{m} \neq m^*\right) d\widehat{m} + \\
& + h_{t-1}^p(m^*) f(m^*-m) \left(1 - \pi_d(m^*, m^*-m)\right) \Bigg) \\
& \eta \left(\int_0^m h_{t-1}^p(\widehat{m}) g(m-\widehat{m}) \mathbb{1}\left(\widehat{m} \neq m^*\right) d\widehat{m} + h_{t-1}^p(m^*) g(m-m^*) \right).
\end{aligned} \tag{14}
$$

$$
h_t^p(m) = \quad \left(1-\pi_a(m)\right) h_t^a(m),
$$

I numerically solve for the stationary distributions $\{h^a(m), h^p(m)\}$ by iterating the above system until convergence. Once that h^a and h^p are known, they can be exploited to compute a large set of statistics that have an empirical counterpart.

Cash management statistics. I start by describing cash management statistics that can be obtained by combining choice probabilities and the stationary distributions. Average cash holdings are given by $M = \frac{M^a + M^p}{2}$, with

$$
M^k = \lim_{\varepsilon \to 0_+} \left(\int_0^{m^*-\varepsilon} m h^k(m) dm + \int_{m^*+\varepsilon}^{\overline{m}} m h^k(m) dm \right) + m^* h^k(m^*), \tag{15}
$$

for $k = a, p$. Withdrawal probabilities during the adjustment and the payment phase are given by

$$
p_w^a = \int_0^{m^*} h^a(m) \pi_a(m) dm, \tag{16}
$$

$$
p_w^p = \lambda \phi_c \left(\int_0^{\overline{m}} h^p(m) \left(1-F(m)\right) dm + h^p(m^*) \left(1-F(m^*)\right) \right), \tag{17}
$$

hence the per-period withdrawal probability is given by $p_w = p_w^a + p_w^p$. I can also compute cash management statistics that are informative about the *size* and *timing* of withdrawals and deposits.

[13] In inventory-theoretical models the stationary distribution is usually obtained as the solution to a Kolmogorov forward equation, a first (or second) order differential equation which characterizes the density function $h(m)$. Given the lumpy nature of the expenditure stream (without any drift or diffusion terms), as well as the discrete-time setup, the KFE boils down to a system of DEs in my setting.

Starting from withdrawals, I need to obtain an expression for the average withdrawal W and for the average amount of cash on hand at withdrawal $\underline{M}$. I do so by computing the stationary distribution of cash holdings conditional on a withdrawal taking place. As withdrawals happen for different reasons during the adjustment and the payment phase, let's denote the conditional density for the former by $h^a(m|w)$ and for the latter by $h^p(m|w)$. Notice that from Bayes' theorem

$$h_w^a(m) = \frac{\Pr_a(a(m) > 0)h^a(m)}{\Pr_a(w)} = \frac{\pi^a(m)\mathbb{1}\left(m < m^*\right)h^a(m)}{p_w^a},$$

and

$$h_w^p(m) = \frac{\Pr_p(a(m) > 0)h^p(m)}{\Pr_p(w)} = \frac{\lambda\phi_c\left(1 - F(m)\right)h^p(m)}{p_w^p}.$$

Notice that $\int_0^{m^*} h_w^a(m)dm = 1$, while h^p has a mass point at m^*, given by $h_w^p(m^*) = 1 - \int_0^{\overline{m}} h_w^p(m)dm$. Recall that the proportions of the two types of withdrawals are given by p_w^a/p_w and p_w^p/p_w. Hence, I can easily compute $\underline{M}$ from

$$\underline{M} = \left(\frac{p_w^a}{p_w}\right)\int_0^{m^*} mh_w^a(m)dm + \left(\frac{p_w^p}{p_w}\right)\left(\int_0^{\overline{m}} mh_w^p(m)dm + h_w^p(m^*)m^*\right), \tag{18}$$

and residually calculate the average withdrawal size W using the fact that $\underline{M} + W = m^*$. I can obtain the same statistics for deposits. Let p_d^a and p_d^p be, respectively, deposit probabilities during the adjustment and payment phase. They are given by

$$p_d^a = \int_{m^*}^{\overline{m}} h^a(m)\pi_a(m)dm, \tag{19}$$

$$p_d^p = \eta\left(\int_0^{\overline{m}} h^p(m)\left(1 - G(\overline{m} - m)\right)dm + h^p(m^*)\left(1 - G(\overline{m} - m^*)\right)\right), \tag{20}$$

while the conditional densities $h_d^a(m)$ and $h_d^p(m)$ are

$$h_d^a(m) = \frac{\pi^a(m)\mathbb{1}\left(m > m^*\right)h^a(m)}{p_d^a}, \quad h_d^p(m) = \frac{\eta\left(1 - G(\overline{m} - m)\right)h^p(m)}{p_d^p}.$$

As before, $\int_0^{m^*} h_d^a(m)dm = 1$, while $h_d^p(m^*) = 1 - \int_0^{\overline{m}} h_d^p(m)dm$. The average cash at deposit is given by

$$\underline{D} = \left(\frac{p_d^a}{p_d}\right) \int_{m^*}^{\overline{m}} m h_d^a(m)dm + \\ \left(\frac{p_d^p}{p_d}\right) \left(\int_0^{\overline{m}} h_d^p(m) \left[\frac{\int_{\overline{m}-m}^{+\infty} (m+r)g(r)dr}{\int_{\overline{m}-m}^{+\infty} g(r)dr} \right] dm + h_d^p(m^*) \left[\frac{\int_{\overline{m}-m^*}^{+\infty} (m^*+r)g(r)dr}{\int_{\overline{m}-m^*}^{+\infty} g(r)dr} \right] \right), \tag{21}$$

and I residually calculate the average deposit size D using the fact that $\underline{D} - D = m^*$.

Payment choice statistics. I now compute some statistics that summarize agents' payment choices which are implied by the set of policy functions. The card share of expenditures γ is given by

$$\gamma = \frac{24\lambda(1-\phi_c)\left(\int_0^{\overline{m}} h^p(m)\gamma(m)dm + h^p(m^*)\gamma(m^*)\right)}{e}, \tag{22}$$

where $\gamma(m) = \int_0^m sf(s)\pi_d(m,s)ds + \int_m^{+\infty} sf(s)ds$ and $e = 24\lambda\mathbb{E}(s)$ is the average total daily expenditure. To gauge how much cards are used to settle *unforced* expenditures, I compute the fraction of purchases paid by cards when visiting a shop of type $\ell = cd$ and when $s \leq m$. I denote this statistic by $\widetilde{\gamma}$, which is given by[14]

$$\widetilde{\gamma} = \frac{24\lambda\phi_{cd}\left(\int_0^{\overline{m}} h^p(m)\left(\int_0^m sf(s)\pi_d(m,s)ds\right)dm + h^p(m^*)\left(\int_0^{m^*} sf(s)\pi_d(m^*,s)ds\right)\right)}{24\lambda\phi_{cd}\left(\int_0^{\overline{m}} h^p(m)\left(\int_0^m sf(s)ds\right)dm + h^p(m^*)\left(\int_0^{m^*} sf(s)ds\right)\right)}, \tag{23}$$

where the denominator yields the total daily expenditure that could be settled using both payment methods, and the numerator denotes the part of this expenditure which ends up being paid for using cards.

4 Estimation

The set of scalar and functional parameters of the model that I need to estimate is given by $\mathbf{\Theta} = \{\beta, R, b, \kappa, \sigma_b, \sigma_\kappa, \psi, F, \lambda, G, \eta, \phi_c, \phi_d\}$. In this Section, I describe the parametrization of the model and its identification.

[14]It is also possible to compute the share of *purchases*, and not expenditure, settled using cards, both overall and when having both options available, as I did in Table 1. I denote these two statistics by γ_n and $\widetilde{\gamma}_n$.

4.1 Parametrization and functional forms

I estimate the model at the hourly frequency, so that multiple payments per day (as observed in the data) are possible. I set the hourly discount factor to $\beta = .991$, so that agents care approximately about discount by 95% cash management and payment costs of two weeks ahead, when making decisions. As already discussed above, I assume that $\epsilon_j \sim \text{EVT1}(-\gamma\sigma_b, \sigma_b)$ for $j \in \{a, n\}$ and that $\chi_k \sim \text{EVT1}(-\gamma\sigma_\kappa, \sigma_\kappa)$ for $k \in \{c, d\}$, where γ is the Euler-Mascheroni constant, so that $\mathbb{E}(\epsilon_j) = \mathbb{E}(\chi_k) = 0$ for any j, k. Finally, I assume that the both the distribution F of payment sizes s and the distribution G of cash inflows r are lognormal, i.e., that $s \sim \text{LN}(\mu_F, \sigma_F^2)$ and $r \sim \text{LN}(\mu_G, \sigma_G^2)$.

4.2 Identification

It is well known (Rust (1994)) that the discount factor is not identified from choice data in dynamic discrete choice models. Therefore, as common in the literature on structural models (Bajari, Benkard, and Levin (2007)), I treat the discount factor as known, and I discuss the identification of the remaining parameters.

Expenditures and cash inflows. I start by discussing the identification of the parameters λ and η, which govern the frequency at which purchases and exogenous cash inflows arise. The parameters λ and η are identified by the reported number of purchases and through information on cash inflows during the diary day, which allow me to elicit the probability of both type of events per unit of time. Under the assumption that agents complete all purchases that arise (i.e., that they perform an unplanned withdrawal when they don't have enough resources to pay), I can identify the four parameters $(\mu_F, \sigma_F, \mu_G, \sigma_G)$ through the empirical distribution of payment sizes and cash inflows, which enable me to pin down the relative frequencies of purchases/inflows of different size.

Merchant acceptance probabilities. I identify the shares of shops that either accept only cash (ϕ_c) or only cards (ϕ_d) through the reported perceived acceptance of payment methods by respondents in each of their purchases. Notice that the implied acceptance probabilities implied by buyers' reports coincide with true acceptance probabilities if buyers do not disproportionately visit shops with a certain acceptance policy. In the model, I assume that agents do not sort into shops based on their acceptance policy but visit stores randomly. Under this assumption, ϕ_c and ϕ_d are identified[15].

Structural parameters. I now discuss the identification of the set of structural preference parameters $\widetilde{\mathbf{\Theta}}_e = \{b, R, \kappa, \sigma_b, \sigma_\kappa, \psi\}$, conditional on all the other parameters being known at this stage. First, I show that $\widetilde{\mathbf{\Theta}}_e$ is not identified, as different values of b yield the same decision rules under an appropriate transformation of the other parameters: only the units of value functions change. The result, which is a version of the well-known result that the adjustment cost b is not identified in inventory theoretical models (Alvarez and Lippi (2009)), is formally stated in Proposition 2 in Appendix B.2.1 and a formal proof is provided there. Decision rules obtained through the model are not informative about the *absolute* magnitude of all parameters in $\widetilde{\mathbf{\Theta}}_e$: however, they do reveal information about the *relative* size of parameters. In particular, I show that the *normalized* parameter vector $\mathbf{\Theta}_e = \left\{\frac{R}{b}, \frac{\kappa}{b}, \frac{\sigma_b}{b}, \frac{\sigma_\kappa}{b}, \psi\right\}$ is indeed identified: it is possible to identify the withdrawal penalty ψ exactly, as well as the ratios between all other parameters and the adjustment cost b. I follow a method proposed by Calvo, Lindenlaub, and Reynoso (2024), who exploit the extreme-value type structure of shocks to link model parameters to choice probabilities. I now discuss the logic behind this identification result, while formal arguments are provided in Appendix B.2 through a series of Lemmas.

The ratio σ_b/b, which determines how large is the scale of disturbances to adjustment decisions compared to the fixed cost of adjusting cash balances, is pinned down by adjustment probabilities for $m \simeq m^*$: since there is no gain from adjusting when $m = m^*$, when $\sigma_b/b \to 0$ there will be no visits to ATMs when m is close to m^*; instead, when the scale of disturbances rises, people might visit ATMs with positive probability when their cash holdings are very close to the optimal level. The ratio σ_κ/κ, which determines how large is the scale of disturbances to payment method choices compared to the fixed cost/benefit of using cards, is identified from card usage probabilities for tiny transactions ($s \to 0$). The reason is that when s is very small, the continuation value for cash and card payments is the same, therefore for these kind of transactions the only things that matter are σ_κ (how large are the shocks to payment decisions) and κ (on average, does people prefer to use cash or cards for payments?). The ratio σ_b/σ_κ, which determines the relative scale of disturbances

[15]The assumption would be problematic if the probability that agents visit a store of type $\ell \in \{c, d, cd\}$ does not reflect their true proportion in the population. This may occur if, for instance, buyers' visit disproportionately shops that accept cards. The assumption might be tested and, possibly, rejected, combining demand-side data on acceptance such as the one I make use of with a supply-side counterpart (e.g., a survey of firms' acceptance decisions). Such a comparison could be helpful to estimate the degree of consumers' search and to determine if the assumption of random sorting is plausible or not.

to adjustment and payment decisions, is identified by comparing payment choice probabilities for two different transactions s and s', when m is the same, scaling them by a discounted difference in adjustment probabilities at $m - s$ and $m - s'$. The intuition is that individuals prefer to use cards to settle transactions that would push them into withdrawing next period with a higher probability. If σ_b is tiny relative to σ_κ, the difference in card usage probability would be much larger than the discounted difference in implied adjustment probabilities, and vice versa. As a result, σ_κ/b and κ/b are identified. The last two parameters, R/b and ψ, are identified by a set of equations linking adjustment and expected payment choice probabilities for two different values of m. The intuition is that a higher m implies higher opportunity costs Rm but also higher insurance value against the payment of ψ. After rescaling for differences in expected future adjustment and payment choice probabilities, and given that all the other parameters are identified at this point, differences in adjustment probabilities for any m and m' are informative about R and ψ.

Proposition 1. *Assume the discount rate $\beta < 1$ is known and that is known and that the parameter set $\mathbf{\Theta}_c = \{\lambda, F, \eta, G, \phi_c, \phi_d\}$ is identified. Then, under the assumed functional forms, the vector of structural parameters $\mathbf{\Theta}_e = \left\{\frac{R}{b}, \frac{\kappa}{b}, \frac{\sigma_b}{b}, \frac{\sigma_\kappa}{b}, \psi\right\}$ is identified.*

Proof. See Appendix B.2.2 ∎

4.3 The data

I estimate model parameters separately for each geographical region in SPACE data. To do that, I collapse the information at the region level, which requires some adjustments that I now discuss. The number of withdrawals in each region is adjusted to take care of measurement error using a strategy described in Appendix A.3, that exploits an accounting identity equating cash inflows and outflows. For each region, I compute the number of reported purchases $\widehat{n}_s$ and the coefficient of variation of their size distribution $\widehat{\mathbb{CV}}_s$. For cash inflows, I proceed slightly differently as I do not observe their number $\widehat{n}_r$ but only whether at least a cash inflow happened during the diary day, i.e., I observe $\widehat{r} = \Pr(n_r \geq 1)$. I still observe the size distribution of inflows and can therefore pin down its coefficient of variation $\widehat{\mathbb{CV}}_r$.

TABLE 2: External calibration: results, average across regions.

Description	Parameter	Avg. value	Source/target
Discount factor	β	0.991	2-weeks horizon
Share accept cash only	ϕ_c	0.162	Reported acceptance
Share accept cards only	ϕ_d	0.064	Reported acceptance
Purchase frequency	λ	0.082	Daily n. purchases $\widehat{n}_s$
Payment size distr., location	μ_F	-1.459	Normalization of daily expenditure ($e = 1$)
Payment size distr., scale	σ_F^2	1.586	$\widehat{\mathbb{CV}}_s$
Inflow frequency	η	0.005	Pr $(\widehat{r} \geq 1)$
Inflow size distr., location	μ_G	-0.506	Daily inflow relative to daily exp. ($i = \hat{i}/\hat{e}$)
Inflow size distr., scale	σ_G^2	1.391	$\widehat{\mathbb{CV}}_r$

Note: The Table displays average values of externally calibrated parameters across the 100 regions in the sample. The discount factor is set so that $\beta^{168 \cdot 2} = 0.05$, where 168 is the number of hours in a week.

4.4 Estimation strategy and results

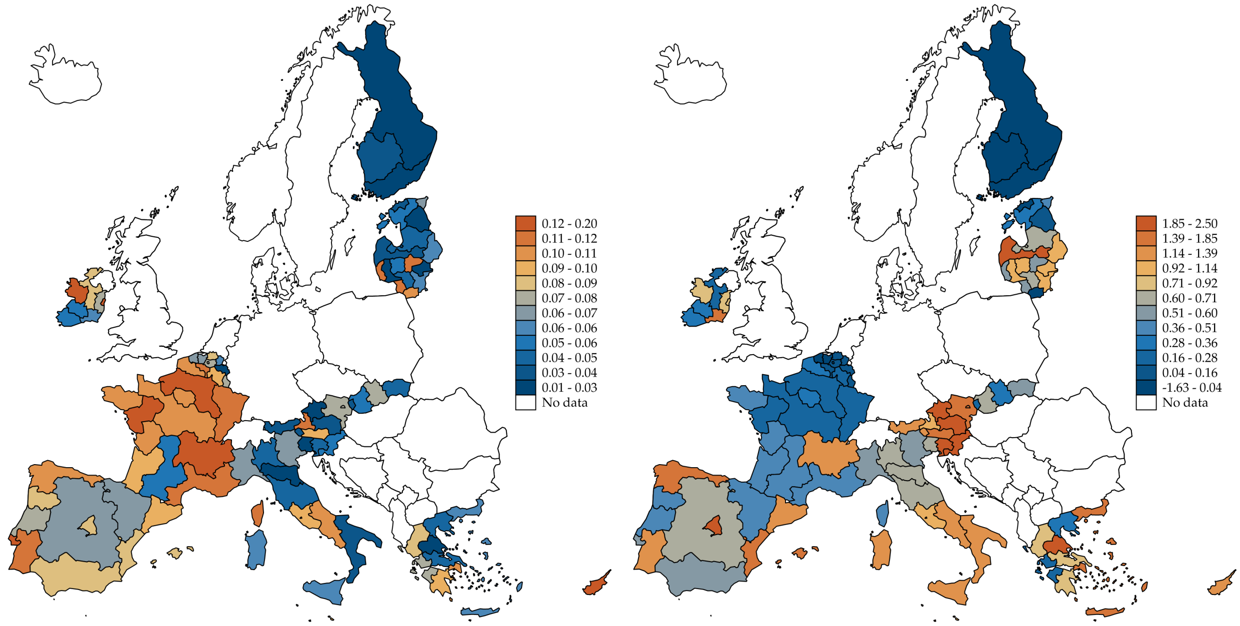
This Subsection describes how I pin down model parameters from the region-level data described above, through two sequential steps: external calibration for parameters that can be directly identified from observables, and minimum distance estimation for the remaining parameters.

4.4.1 Step I: External calibration

In the first step of the estimation procedure, I calibrate the parameters that govern the size and frequency of expenditures (μ_F, σ_F^2 and λ), the size and frequency of cash inflows (μ_G, σ_G^2 and η), and merchant acceptance probabilities (ϕ_c and ϕ_d). The hourly probability of a purchase λ is calibrated to match the observed daily number of payments $\widehat{n}_s$, assuming that $n_s \sim \text{Binomial}(24, \lambda)$. The hourly probability of a cash inflow η is calibrated to match the share of respondents $\widehat{r}$ that had at least one cash inflows by assuming that $n_r \sim \text{Binomial}(24, \eta)$, which yields $\Pr(n_r \geq 1) = \widehat{r} = 1 - (1-\eta)^{24}$, enabling me to pin down η. I then obtain (μ_F, σ_F^2) and (μ_G, σ_G^2) by i) normalizing daily expenditures $e = 24\lambda \cdot \mathbb{E}(s)$ to one, ii) normalizing daily cash inflows to $i = \hat{i}/\hat{e}$, the observed ratio of daily inflows to daily expenditure, and iii) by matching the coefficients of variation of the size distributions of expenditures and inflows - see Appendix B.1 for details. Finally, ϕ_c and ϕ_d are directly calibrated by matching the relevant empirical counterparts.

FIGURE 9: Estimated parameters $\widehat{\mathbf{\Theta}}_e$ - part I.

(A) Yearly opportunity cost R (B) Card usage cost/benefit κ/b



Note: In the left panel, I plot estimated opportunity cost of using cash R, expressed in yearly terms through the formula $R = (1 + R_h)^{8760} - 1$, where R_h is the estimated opportunity cost at the model frequency (one hour) and 8760 is the number of hours in a year. In the right panel, I plot the estimated fixed cost/benefit of using cards κ, normalized by the level of withdrawal costs b. The level of geographic aggregation is the NUTS-1/2/3 region level (100 regions in total). In both panels, regions are split in 12 quantiles and colored according to the quantile they belong to.

4.4.2 Step II: Internal estimation

I estimate the parameter set $\mathbf{\Theta}_e$ through a minimum distance procedure. First, I normalize b to 0.00005, in order to target a yearly opportunity cost of the same order of magnitude of the yearly interest rate. This normalization exploits the standard BT model as a benchmark, as outlined in Lippi and Moracci (2024). I then estimate the parameter vector $\mathbf{\Theta}_e = \{R, \kappa, \sigma_b, \sigma_\kappa, \psi\}$ to match five model implied moments and their empirical counterparts. I select two moments that summarize cash management decisions and three statistics that describe payment method choices by households. The two cash management moments are average cash balances normalized by expenditure, M/e, and the yearly number of withdrawals n. As for payment choices moments, I use the overall share of expenditures paid for using cards γ, and the shares of expenditure ($\widetilde{\gamma}$) and purchases ($\widetilde{\gamma}_n$) paid for using cards when having both options possible (unforced payment choices).

The choice of moments is guided from identification arguments, even though all parameters affect all moments. First, I choose M/e as it is particularly informative about the opportunity cost of holding cash R: as holding cash gets costlier, the means of the stationary distributions of cash balances h^a and h^p shifts to the left. I pick n as it contains information about the scale of random disturbances to adjustment decisions σ_b: for given b, indeed, the number of withdrawals per year is increasing in σ_b. It is particularly important to include both $\widetilde{\gamma}$ and $\widetilde{\gamma}_n$, the card shares of expenditure and transactions when facing unforced purchases. Indeed, as I need to identify both κ (the fixed cost/benefit of card payments with respect to cash ones) and the scale of disturbances to payment choices σ_κ, I need two moments which help us to disentangle these two parameters. First, I need $\widetilde{\gamma}$ since it is informative about κ: when $\kappa < 0$, if $\sigma_\kappa \to 0$, then $\widetilde{\gamma} \to 1$, as agents would settle any transaction with cards if they had the chance; at the same time, with $\sigma_\kappa \to 0$, $\widetilde{\gamma} \to 0$ as $\kappa \to b$ (see Proposition 1 in Lippi and Moracci (2024)). Moreover, when $\sigma_\kappa \to 0$, and $\kappa > 0$, the theory predicts that the card share of unforced expenditure $\widetilde{\gamma}$ is larger than the card share of unforced purchases $\widetilde{\gamma}_n$, since agents disproportionately settle large transactions using cards to avoid remaining with little cash on hand after a transaction, for precautionary motives[16]. As σ_κ rises, the shares $\widetilde{\gamma}$ and $\widetilde{\gamma}_n$ get closer: hence, this parameter is identified by the difference between the two shares, while κ is identified from the level of $\widetilde{\gamma}$. These identification arguments make particularly clear why it is crucial to be able to observe the payment choice set of agents (through the level of cash on hand and information on merchant acceptance), in order to compute statistics on unforced transactions which are informative about preference parameters and the relevance of unmodeled features of the problem. Finally, I include the overall share of expenditures γ, as the difference between γ and $\widetilde{\gamma}$ (usually positive in the data) is informative about how frequent are situations in which cards are employed because cash was not sufficient, which helps me pinning down ψ: when ψ is high, agents visit ATMs when they still have lots of cash on hand and thereby forced card payments happen less often.

The model parameters are estimated at the region level, with the level of geographic aggregation (NUTS-1, NUTS-2 or NUTS-3) depends on how geographical information is provided in SPACE for each country. Due to the high amount of parameters estimated (5 parameters $\times$ 100 regions), I display my estimates using maps, while in Figure C.1 in the Appendix I display country-level parameter estimates obtained through population-weighted means of region-level estimates. In Figure 9 I plot the estimated values of R (in yearly terms) and κ (relative to the adjustment cost b).

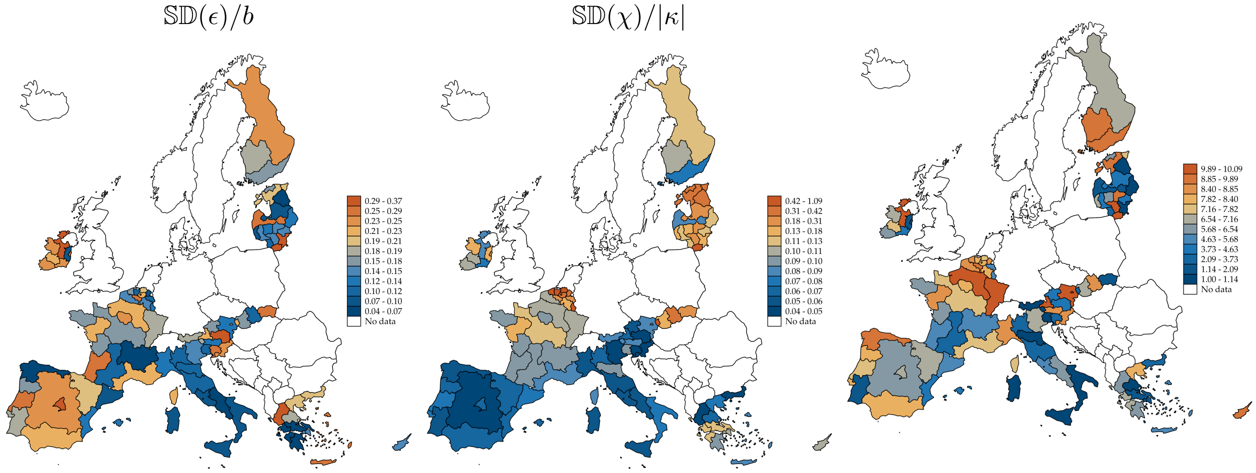
[16]See Lippi and Moracci (2024) for this theoretical result. Even if in that model the precautionary motive stems from missed purchase opportunities and not from the forced withdrawal penalty ψ, the logic still applies.

FIGURE 10: Estimated parameters $\widehat{\Theta}_e$ - part II.

(A) Adjustment shock rel. scale $\mathbb{SD}(\epsilon)/b$

(B) Payment shock rel. scale $\mathbb{SD}(\chi)/|\kappa|$

(C) Withdrawal penalty ψ



Note: In the left panel, I plot the estimated standard deviation of the adjustment shock relative to b, with $\mathbb{SD}(\varepsilon) = \sqrt{\pi^2\sigma_b^2/6}$ being the standard deviation of a Gumbel distribution with scale parameter σ_b. Similarly, $\mathbb{SD}(\chi) = \sqrt{\pi^2\sigma_\kappa^2/6}$ for the middle panel, where I display the estimated standard deviation of payment choice shocks relative to the absolute value of κ. Finally, in the right panel I display the distribution of the estimated withdrawal penalty ψ. The level of geographic aggregation is the NUTS-1/2/3 region level (100 regions in total). In all panels, regions are split in 12 quantiles and colored according to the quantile they belong to.

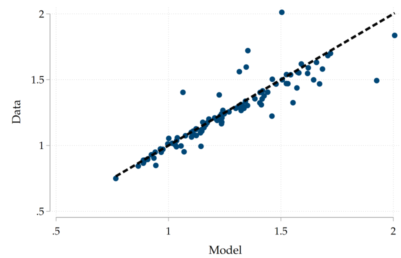
My estimates for R are particularly high for parts of France, Greece, Ireland, Portugal and Spain (higher than 10% and sometimes as high as 20% in yearly terms), while they are much smaller and sometimes as low as 1-3% for areas of Austria, Estonia, Finland and Italy. The estimated level of κ/b lies between 0 and 1 for most regions, albeit there are some regions in which the estimated κ is larger than withdrawal costs b (mainly in areas of Spain and Austria), while on the other hand in large parts of Belgium and Finland I estimate a negative value of κ, meaning that on average people from those regions prefer card payments to cash ones. As for the extent of random disturbances, I use the following method to quantify how much factors outside the model are relevant in explaining the observed variation in agents' choices, compared to modeled ones. First, I compute the standard deviation of the EVT1 distributions implied by each estimate of σ_b and σ_κ, to recover the standard deviation of shocks ε to adjustment decisions and of shocks χ to payment choices. Finally, I compare these standard deviations respectively with b and with the absolute value of κ, to measure how large are the estimated shocks compared to modeled fixed costs/benefits of adjustments/card usage decisions. In terms of the scale of adjustment shocks, results show that shocks to withdrawal choices are smaller (about half as large) than disturbances to payment choices. This suggests that the determinants of adjustment choices which are explicitly modeled (state dependence on the level of cash balances m) explain a larger portion of withdrawal and deposit behavior compared to the amount of variation in payment choices captured by the state variables m and s. In other words, the extent to which unmodeled features of the problem impact households' choices is larger for payment method than for adjustment decisions[17]. In geographical terms, as I show in Figure 10, shocks to adjustment decisions play a relevant role in Ireland and Slovenia in particular, while shocks to payment choices are a major component of decisions in Belgium. In two countries, Greece and Italy, both types of shocks play a very minor role, suggesting that the model accurately describes households' behavior in such areas. Finally, the estimated penalty for forced withdrawals ψ exhibits a large amount of variation across the Euro Area, ranging from almost no penalty in areas of Greece, Italy, Slovenia and Spain, to large penalties in Finland, France, and parts of Ireland, Portugal and Slovakia. In Figure C.3 in the Appendix, I also show a decomposition of the variance of estimated parameters across the 100 regions in a within-country and a between-country component. Results show that most of the overall variation in σ_b and ψ (around 70-80%) is due to within-country variation, while for σ_κ the within and between variance shares are approximately equal. For R and κ cross-country differences play a larger role than regional ones: respectively, for these two parameters the between share of variance accounts for 55% and 73%.

I now discuss how well does the model is able to fit targeted moments. In Figure 11 I compare, for each targeted moment, its model-implied value when parameters are set to their estimated value and its empirical counterpart. I overlay the 45 degree line to each plot: a perfect fit would see all points on this line. It is possible to see that the model does remarkably well well in replicating targeted features of cash management and payment behavior, simultaneously capturing the most salient features of cash management and payment choices by households. One moment, the overall card share of expenditures γ is much more frequently underestimated than overestimated. Despite this, for a large number of regions the fit is pretty good (the median percentage deviation of model-implied moments from their empirical counterparts is around 5%), as illustrated by Figure C.2 in the Appendix.

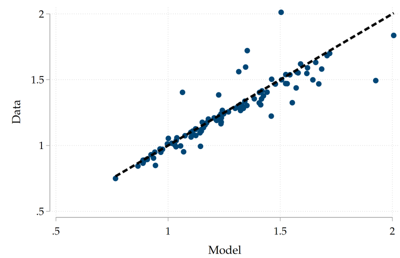
[17] A possible interpretation is that payment method choices also involve other actors, such as the merchant. Even when the store accepts cards, sellers' preferences might still have an impact on buyers' choices. Figure A.7 in the Appendix of Lippi and Moracci (2024) suggests that this is indeed the case: 20% of respondents to the 2016 Survey on the Use of Cash by Households suggest that seller preferences play a role in their payment method choices.

Figure 11: Model fit to targeted moments.

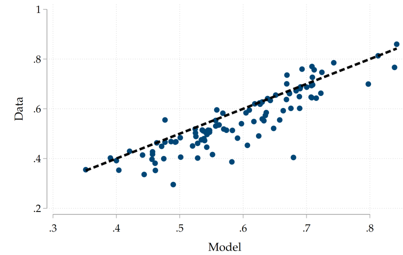
(A) Normalized average cash holdings M/e



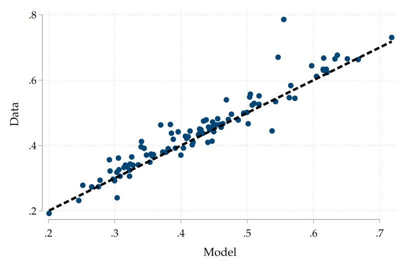
(B) Average number of withdrawals n



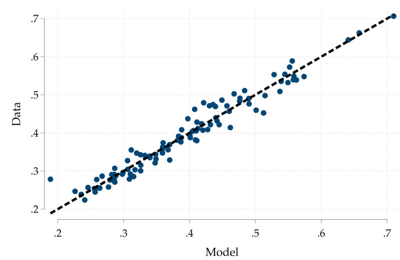
(C) Card share of expenditures γ



(D) Card share of unforced expenditures $\widetilde{\gamma}$



(E) Card share of unforced purchases $\widetilde{\gamma}_n$



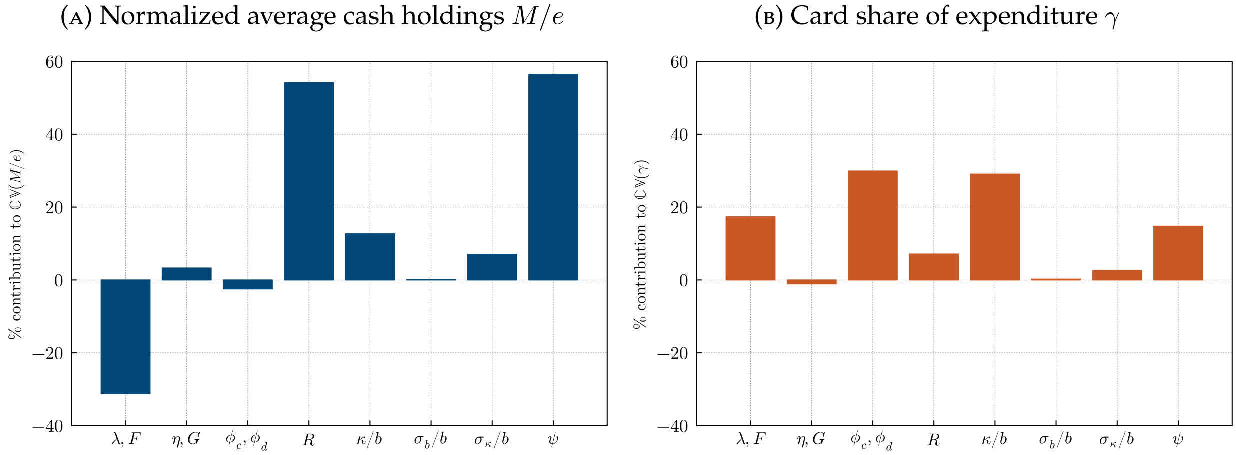
Note: This Figure illustrates how well the model fits targeted moments, across the 100 regions my estimation sample. For each moment, I display the model-implied moment (at the estimated parameter values) and its empirical counterpart. A perfect fit would see all points lying on the 45 degree line, which I overlay to each plot.

5 Sources of Euro Area differences

I now use the estimated model to decompose differences in cash holdings and the intensity of card usage across Euro Area regions into their determinants. The model enables me to elicit the contribution of several factors in generating regional differences, including i) differences in the frequency and size of payments/inflows, ii) difference in card and cash acceptance rates and iii) differences in structural parameters. I do that by decomposing the model-implied coefficients of variation of M/e (.20) and γ (.18) into its components[18]. I do that by running counterfactual experiments in which I in turn replace each parameter (or pair of parameters, in case of (λ, F) and (η, G)) with its Euro Area mean obtained through a population weighted averaging of estimated/calibrated parameters for all regions. For what concerns average cash holdings, results reveal that the key determinants of regional heterogeneity across the Euro Area are differences in opportunity costs R and in the penalty associated with unforced withdrawals ψ, which can be though of as a measure

[18]It is worth mentioning that the coefficients of variation for M/e and γ computed on the cross-section of model-implied moments (for the estimated value of parameters) closely resemble those reported in Table 1.

Figure 12: Sources of regional differences in cash management and payment choices.



Note: In this Figure, I show the relative contribution of a number of factors to the observed variation in M/e and γ across the 100 regions in my sample, through a decomposition of the coefficient of variation of M/e (left panel) and γ (right panel). An estimated negative contribution of one parameter means that if such parameter was replaced with its overall mean, the coefficient of variation across regions would increase.

of ATM density, as higher values of ψ imply larger costs for unplanned withdrawal needs. Also differences in the level of κ and in the scale of payment shocks σ_κ matter, even though to a minor extent. Differences in exogenous cash inflows have a small positive impact on aggregate variation. The contribution of differences in the size and frequency of payments is negative, meaning that differences in expenditure patterns actually end up equalizing M/e across regions. As for the determinants of the observed regional differences in the card share of expenditure γ, the estimated model reveals that the bulk of differences across areas stem from heterogeneity in acceptance rates (which explains 30% of it) and in preferences for cash versus cards (another 30%). Differences in the frequency and size of payments, opportunity costs and withdrawal costs also explain substantial shares of the coefficient of variation of γ (respectively 17, 8 and 15%), while the contributions of other factors are negligible. These results suggest that even under a convergence in the extent of supply-side constraints to card or cash usage (which looks plausible given the trends in card acceptance rates, see Lippi and Moracci (2024)), differences in payment choices around the Euro Area are there to stay, as they reflect fundamental heterogeneity in the context where individuals live (ψ, λ), in their shopping behavior (λ, F) and most importantly in their preferences (κ).

6 Conclusion

In this paper, I combined data and theory to study the sources of such differences through a structural model. Drawing upon data from the Study on the Payment Attitudes of Consumers in the Euro Area (SPACE), an ECB survey ran in 17 countries in the years 2021-22, I provided evidence on the magnitude of cross-regional variation in cash holdings and in the share of expenditure settled using cards between and within EA countries. I then exploited the detailed information collected in SPACE, to show that differences in average cash held emerge as a result of heterogeneity in withdrawal policies (size, frequency and timing of cash replenishments) and in the amount of cash received from others. At the same time, I showed that the variation in the intensity of card usage can be attributed to a mix of supply-side and demand-side factors: on one hand, cards are accepted much more frequently in some regions than in others (and even cash is not uniformly accepted across different regions); on the other, even when they have the option to use cards, households employ them with very different probabilities across areas, and the propensity to do so correlates with reported preferences for cards versus cash as a means of payment.

To elicit the relative importance of all these factors in shaping the geographical distribution of payment and cash management choices by Euro Area households, I build a novel quantitative model that combines feature of the classical cash management model by Baumol (1952) and Tobin (1956), endogenous payment method choices as in Alvarez and Lippi (2017), a stochastic stream of lumpy expenditures and cash inflows with a realistic size distribution, imperfect acceptance of cash and cards by merchants. All these features of the model are disciplined through data from payment diaries, either through external calibration or minimum distance estimation. By including extreme-value distributed random disturbances to the dynamic discrete choices faced by agents (adjustment and payment method decisions), I am also able to provide an identification result for the vector of structural parameters.

The estimation results reveal that in the majority of Euro Area regions, cards are more costly to use than cash for most households. They also show the existence of large opportunity costs of holding cash in parts of the Euro Area. Most of the variation in estimated parameters is within countries, with the exception of preferences for cards versus cash as a means of payment, which are highly homogeneous within each country and exhibit large variation across countries. I perform a decomposition of the cross-regional coefficient of variation of cash holdings into its determinants, showing that i) most differences in average cash holdings are explained from heterogeneity in the opportunity cost of holding cash and in the cost of accessing cash, and ii) the bulk of differences in the intensity of card usage are due to variation in supply-side constraints (30%), preferences (30%), with the frequency and size of expenditures, opportunity and withdrawal costs playing a smaller, but non-negligible role. My results suggest that only a small portion of differences across regions and countries are due to the imperfect acceptance of cards by merchants, and that even under a plausible convergence in card acceptance rates, geographical differences in cash holdings and card usage will persist as a result of heterogeneity in preferences, cash holding and withdrawal cost across different Euro Area regions.

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A Empirical appendix

A.1 The focus on area disparities

In this Section, I provide evidence showing that it is worth investigating heterogeneity by area. As there are many dimensions of heterogeneity (mainly demographics) which could be explored, I need to argue that geographical area is a relevant one. An objection could be that variability across areas simply reflects the variation in demographic composition (and therefore preferences) between areas, while area itself does not contribute to the observed variation. In other words, if we were to equalize the demographic composition (in terms of education, age, income and so on) across geographical areas, we would see no differences anymore. I loosely borrow from Gibbons, Overman, and Pelkonen (2014), that develop an empirical methodology to evaluate the relevance of people vs place in determining area disparities. I estimate regression models of the type

$$y_i = \boldsymbol{x}_i'\boldsymbol{\beta} + \boldsymbol{d}_i'\boldsymbol{\gamma} + \varepsilon_i, \tag{24}$$

where y_i are (i) cash at the start of the diary day for individual i (normalized by average expenditure in the region/year), and (ii) card share of expenditure for individual i, while $\boldsymbol{x}_i$ is a vector of covariates about i (education, age, sex, survey year, and additional controls) and $\boldsymbol{d}_i$ is a $L \times 1$ vector (where L is the number of distinct geographical areas) whose ℓth component is equal to one if individual i comes from region ℓ. For each of the two outcomes, I run four specifications, varying the set of controls and the level of geographical aggregation (country or NUTS-2 level region). Then, I run a regression-based decomposition of the coefficient of variation following Cowell and Fiorio (2011), which yields three outputs. First, it shows that the portion of total variation captured by model (24) is extremely small (2.5 to 5%) for cash holdings, while it is larger (10 to 20%) for the card share of expenditure. This shows that the variation *within* each demographic and geographical category is larger than variation between categories, especially for cash holdings. Second, it shows that, even after controlling for a large set of demographic characteristics and on reported preferences for payment methods, geographical area accounts for more than 25% of the explained variation in cash balances, and for around 35% of the explained variation in the card share of expenditure. In other words, area is a strong predictor of behavior: people with similar

Table 3: Contribution of geography and demographics to observed variation in cash holdings.

Dep. variable	Normalized cash balances				Card share of expenditure			
	Geographical aggregation level							
	Country	NUTS2	Country	NUTS2	Country	NUTS2	Country	NUTS2
Explained variation (%)	2.73	3.06	4.59	4.91	11.21	11.45	19.17	19.38
Education	1.16	0.65	0.33	0.18	5.16	4.68	1.43	1.35
Age	38.29	28.65	21.11	15.25	1.30	1.17	1.22	1.26
Sex	14.87	15.73	6.75	7.12	0.08	0.06	0.00	0.00
Year	16.70	16.69	20.62	19.44	29.33	27.83	12.09	11.91
Geography	28.98	38.28	20.58	26.07	64.13	66.27	31.49	33.21
Employment	·	·	4.79	7.90	·	·	1.75	1.63
Income	·	·	4.63	4.19	·	·	3.71	3.26
Household size	·	·	0.28	0.11	·	·	0.33	0.29
Preferences	·	·	20.92	19.75	·	·	47.98	47.08
Residual variation (%)	97.27	96.94	95.41	95.09	88.79	88.55	80.83	80.62
Total variation (%)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Figure A.1: Violation of the BT accounting identity.

demographic characteristics behave differently (in terms of cash management and payment choice) if they live in different geographical areas.

A.2 An accounting identity

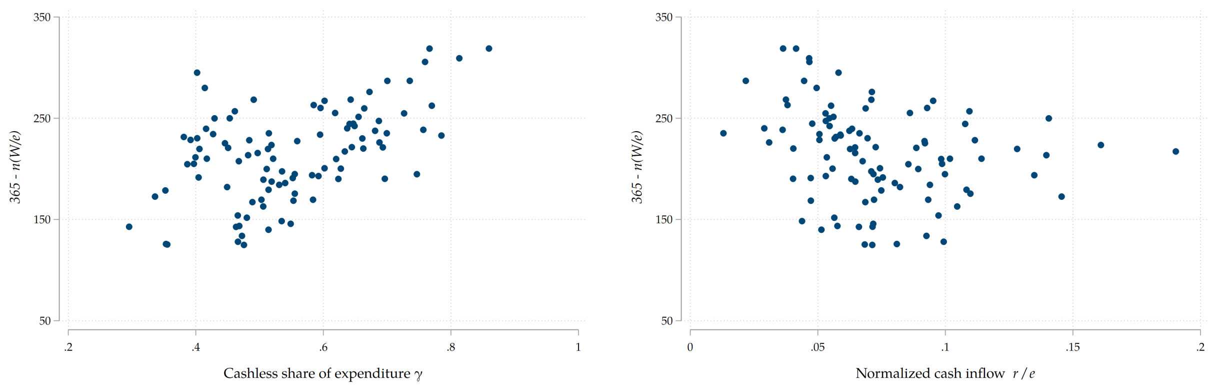
In the BT model of cash management and in many other models (Alvarez and Lippi (2009), Alvarez and Lippi (2013)), the fundamental accounting identity

$$nW = 365e,$$

holds, where n is the yearly number of withdrawals and e are average daily expenditures, to stick with the notation of this paper. Indeed, in such frameworks, expenditures can be only financed through cash withdrawals. As I show in Figure A.1, the above identity is empirically violated by a large margin. The Figure displays the histogram of days of expenditure per year that are not financed through cash withdrawal, i.e., I plot $365 - n(W/e)$, which should be equal to zero from the BT identity. Each observation is a region. On average, more than 200 days of expenditure per year are not financed through cash withdrawals. There are two main reasons for this fact. First, a share γ of expenditures is financed using cards, hence the total yearly expenditure which must be

Figure A.2: Explaining the violation of the BT accounting identity.

(a) Card payments (b) Cash inflows



financed using cash is given by $c = (1-\gamma)365e$. This is taken into account in models that allow for part of expenditures to be paid in cards, such as Alvarez and Lippi (2017) and Lippi and Moracci (2024). In Figure A.2a, I show how the number of expenditure that can't be financed through cash withdrawal is higher where the share of expenditure settled using cards is sizeable. Second, individuals exogenously receive (without the need to visit ATMs) an amount r of cash per year, as cash income or from peer-to-peer transactions. In Figure A.2b, I that as the amount received (relative to daily expenditures e) rises, the violation of the identity gets smaller. This evidence is compatible with a generalized version of the BT accounting identity, which (under the assumption that agents do not make any cash deposits, which is not necessarily true, given that they receive some cash exogenously), is given by

$$nW + r = (1-\gamma)365e. \tag{25}$$

A.3 Deposits and withdrawals: using the accounting identity

If we allow for a positive amount of cash deposited per year d, the accounting identity becomes

$$(1-\gamma)365e + d = nW + r, \tag{26}$$

TABLE 4: Accounting identity and adjustment of n.

	Year		
	2021	2022	Total
Daily expenditure e (EUR)	60.25	62.54	61.40
Card share of purchases γ	0.56	0.58	0.57
Average withdrawal W (EUR)	94.44	92.64	93.54
N. withdrawals per year n	87.54	92.25	89.90
Amount received per year $n_r r$ (EUR)	1455.79	1675.50	1565.65
$(nW + n_r r)/(1-\gamma)365e$	1.01	1.06	1.03
Implied amount deposited per year $n_D D$ (EUR)	114.39	540.25	327.32

Note: We compute the implied number of withdrawals as $\widetilde{n} = \frac{(1-\gamma)365e-r}{W}$.

stating that the average cash outflow (given by the sum of cash-settled expenditures $c = (1-\gamma)365e$ and cash deposits d) in a certain period of time must be equal to the average cash inflow (cash withdrawn nW plus cash received r) in the same period of time. SUCH and SPACE data do not provide direct information about cash deposits, therefore we cannot observe d[19]. We assume that individuals deposit less cash than the amount they received from others, i.e., that $d \in [0, r]$, which seems plausible as they would not deposit cash which was withdrawn previously. Notice that by (26), we must have that $nW + r \geq (1-\gamma)365e$, as even in absence of deposits, all cash expenditures must be financed either through cash withdrawals or cash inflows. Table 4 shows that indeed cash inflows are around 3% larger than cash expenditures[20]. Their difference corresponds to the average amount deposited by each households in a year, which is rather small (around 330 euros). Due to measurement error, the accounting identity fails to hold for every single region. Therefore, we adjust n for each region such that $(nW + n_r r)/(1-\gamma)365e = 1.03$, i.e., we set $n = \frac{1.03(1-\gamma)365e-r}{W}$. For some regions, due to small sample sizes, setting n using the above formula resulted in an implausibly high or low number of yearly withdrawals, due to an unrealistic value for W. In such cases, I adjusted W computed the 95% confidence interval for W using the standard error of the mean, and set it equal to the bound of the CI closer to the national average, before applying the formula for n.

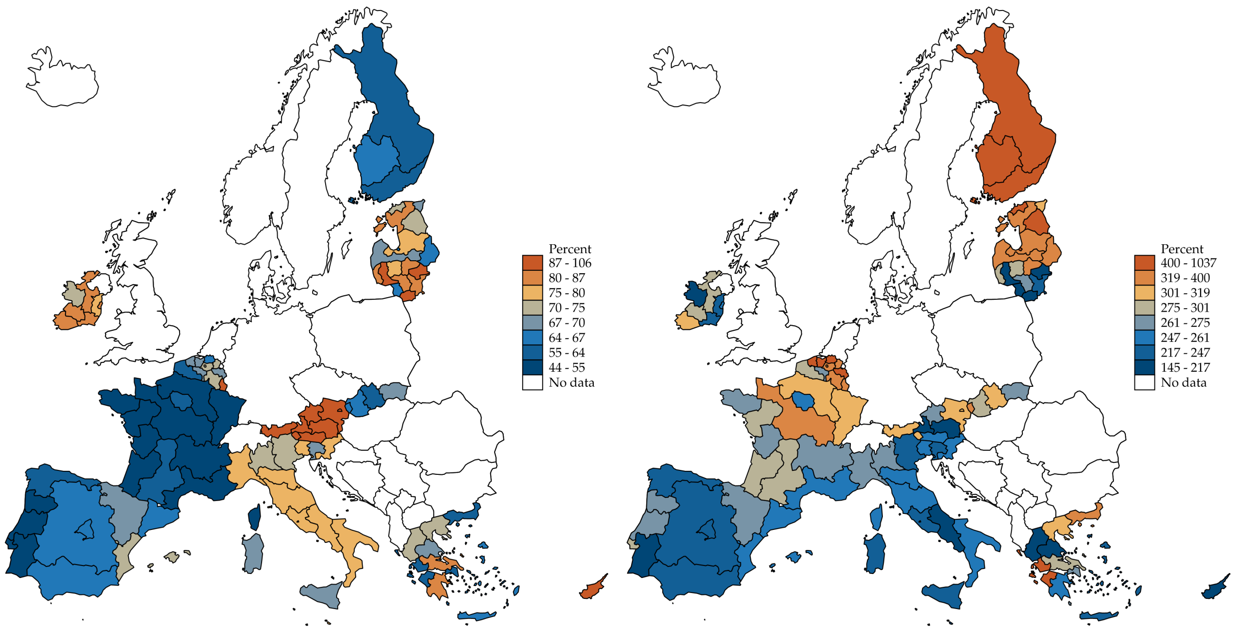
[19]The question "From the cash you had in your wallet, purse or pockets, how much did you put aside during the day?" was asked, but the amount of nonzero responses suggests that respondents interpreted it as a question about savings or cash reserves at home, and not about cash deposits.

[20]I could have assumed that $d = 0$ and simply set $n = \frac{(1-\gamma)365e-r}{W}$. Instead, I decided to let the data speak and allow for a nonzero amount of yearly deposits, as in my stochastic model some deposits will arise as a consequence of large exogenous cash inflows.

A.4 Additional figures

FIGURE A.3: Alternative measures of cash holdings.

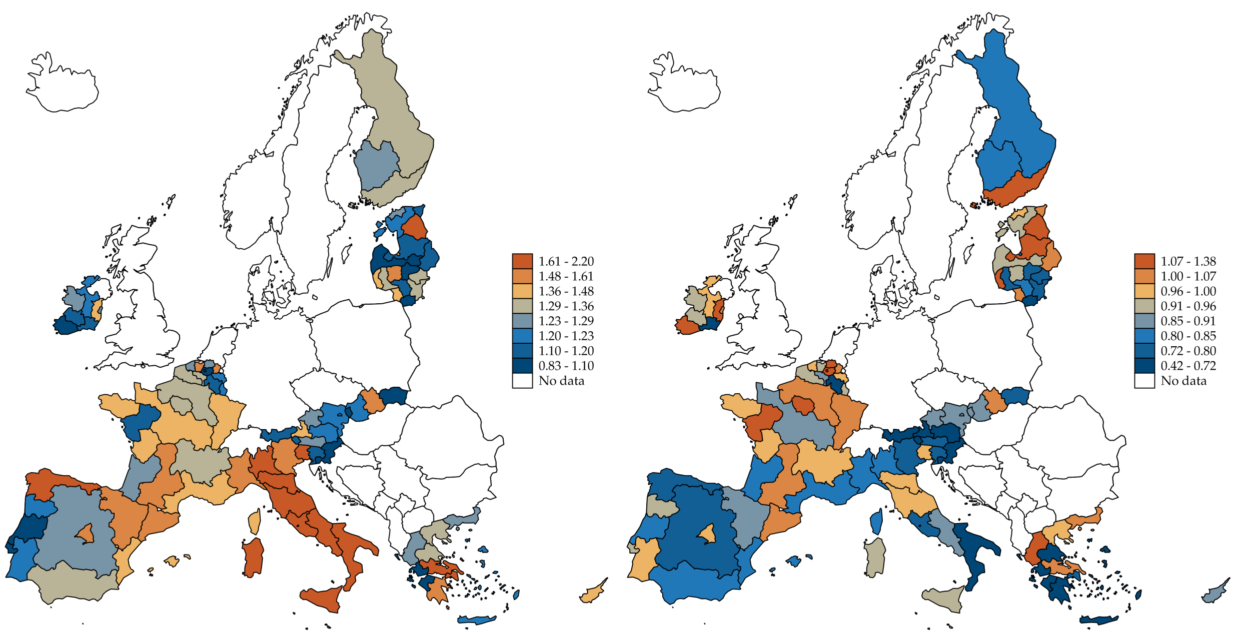
(A) Cash holdings M (EUR) (B) Normalized cash holdings M/c



Note: In the left panel, I plot the average cash balances reported during the dairy day by each respondent. In the right panel, I plot normalized cash balances M/c, i.e., average cash holdings M divided by daily cash expenditure c. The level of geographic aggregation is the NUTS-2 region level (100 regions in total). In both panels, regions are split in eight quantiles and colored according to the quantile they belong to.
Data source: Study on the Payment Attitudes of Consumers in the Euro Area, ECB (Wave II, 2021-22)..

Figure A.4: Average withdrawal and cash at withdrawal relative to M.

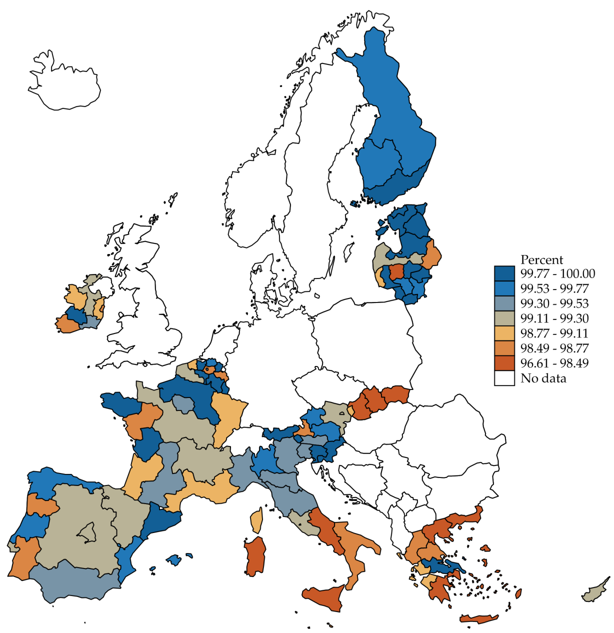
(a) Withdrawal size W/M (b) Cash at withdrawal $\underline{M}/M$



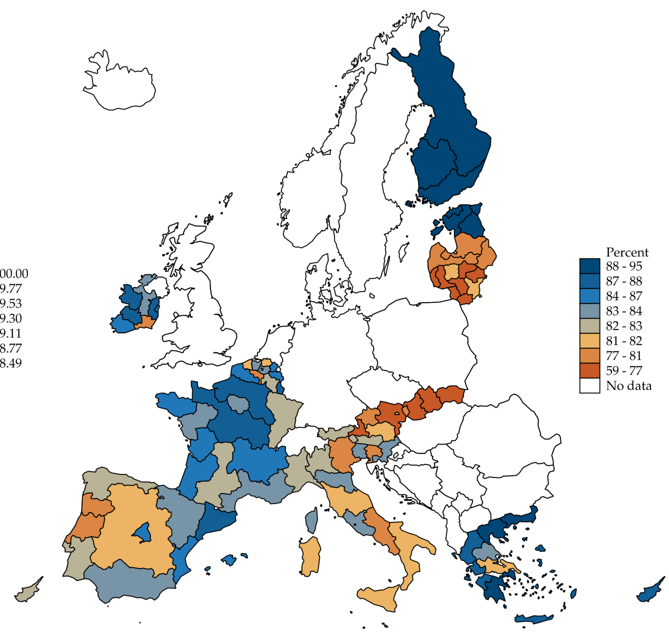
Note: This Figure complements Figure 2a and Figure 3a, displaying a different normalization for W and $\underline{M}$, i.e., the ratios W/M and $\underline{M}/M$.
Data source: Study on the Payment Attitudes of Consumers in the Euro Area, ECB (Wave II, 2021-22).

Figure A.5: Demand and supply side-constraints to payment choices, detail.

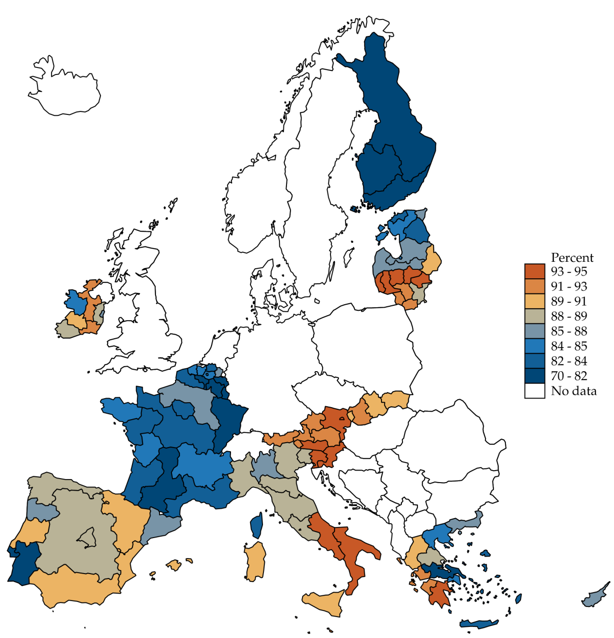
(a) Access to cashless methods (% households)



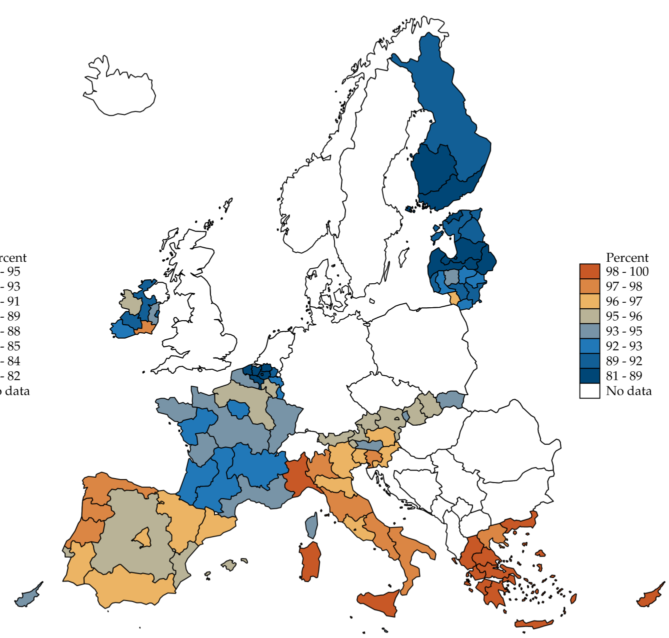
(b) Cashless accepted (% purchases)



(c) Cash sufficient (% purchases)



(d) Cash accepted (% purchases)



B Estimation

B.1 Calibration of expenditure stream and cash inflows

In this Subsection, I describe in detail how to estimate the externally calibrated parameters.

Expenditure stream. Let's start from the calibration of the expenditure stream. Recall that total observed daily expenditure is given by

$$\widehat{e} = \widehat{n}_s \cdot \widehat{\mathbb{E}}(s), \tag{27}$$

where $\widehat{n_s}$ is the average daily number of payments and $\widehat{\mathbb{E}}(s)$ is the average size of a payment. In the model, daily expenditure is given by

$$e = 24\lambda \int_0^\infty s dF(s) = 24\lambda \cdot \mathbb{E}(s). \tag{28}$$

We immediately obtain $\lambda = \widehat{n}_s/24$. We want total daily expenditure in the model to be equal to 1. Therefore, we immediately obtain that

$$\mathbb{E}(s) = \frac{1}{24\lambda} \tag{29}$$

Transaction sizes in the model are distributed as $s \sim \mathrm{LogNormal}(\mu_F, \sigma_F^2)$; their cdf and pdf are respectively denoted by F and f. From (29) we obtain a first restriction on (μ_F, σ_F), i.e.,

$$\mathbb{E}(s) = \exp\left(\mu_F + \frac{\sigma_F^2}{2}\right) = \frac{1}{24\lambda} \ \Rightarrow\ \mu_F + \frac{\sigma_F^2}{2} = -\log(24\lambda). \tag{30}$$

As a second restriction, we want our payment size distribution in the model to have the same coefficient of variation as the observed one. Let $\widehat{\mathbb{CV}}_s = \widehat{\sigma}_s/\widehat{\mathbb{E}}(s)$ denote the observed coefficient of variation of payments. Our second restriction on parameters is then given by

$$\mathbb{CV}(s) = \frac{\sqrt{\mathbb{V}\mathrm{ar}(s)}}{\mathbb{E}(s)} = \frac{\sqrt{\left(\exp\left(\sigma_F^2\right) - 1\right)\exp\left(2\mu_F + \sigma_F^2\right)}}{\exp\left(\mu_F + \frac{\sigma_F^2}{2}\right)} = \widehat{\mathbb{CV}}_s, \tag{31}$$

which implies

$$\frac{\sqrt{\left(\exp(\sigma_F^2)-1\right)\left[\exp\left(\mu_F+\frac{\sigma_F^2}{2}\right)\right]^2}}{\exp\left(\mu_F+\frac{\sigma_F^2}{2}\right)} = \widehat{\mathbb{CV}}_s, \quad \Rightarrow \quad \sqrt{\left(\exp\left(\sigma_F^2\right)-1\right)} = \widehat{\mathbb{CV}}_s$$

$$\Rightarrow \quad \exp\left(\sigma_F^2\right) - 1 = \widehat{\mathbb{CV}}_s^2$$

$$\Rightarrow \quad \exp\left(\sigma_F^2\right) = \widehat{\mathbb{CV}}_s^2 + 1$$

$$\Rightarrow \quad \sigma_F^2 = \log\left(\widehat{\mathbb{CV}}_s^2 + 1\right)$$

and finally yields the calibrated scale and location parameters of F, i.e.,

$$\begin{aligned} \sigma_F &= \sqrt{\log\left(\widehat{\mathbb{CV}}_s^2 + 1\right)}, \\ \mu_F &= -\log(24\lambda) - \frac{\log\left(\widehat{\mathbb{CV}}_s^2 + 1\right)}{2}. \end{aligned} \tag{32}$$

Exogenous cash inflows. Now I deal with the frequency and distribution of cash inflows. Let $\widehat{i} = \widehat{n}_r \widehat{\mathbb{E}}(r)$ denote the total observed daily cash inflow. In the model, the total daily cash inflow is given by

$$i = 24\eta \int_0^\infty r dF(r) = 24\eta \cdot \mathbb{E}(r). \tag{33}$$

Unfortunately, we do not observe the number of daily cash inflows $\widehat{n}_r$, but we only observe whether there is at least one cash inflow during the day. Let $\Pr(n_r \geq 1) = \hat{r}$ be the probability that there is at least one cash inflow. Notice that if the hourly probability of a cash inflow is η, then $\Pr(n_r = 0) = (1-\eta)^{24}$ and then $\Pr(n_r \geq 1) = \hat{r} = 1 - (1-\eta)^{24}$, which yields $\eta = 1 - \sqrt[24]{1-\hat{r}}$. Then, we set $i = \widetilde{i} = \bar{i}/\bar{e}$, and we get

$$\mathbb{E}(r) = \frac{\widetilde{i}}{24\eta} \tag{34}$$

Cash inflows in the model are distributed as $r \sim \mathrm{LogNormal}(\mu_G, \sigma_G^2)$; their cdf and pdf are respectively denoted by G and g. From (34) we obtain a first restriction on (μ_G, σ_G), i.e.,

$$\mathbb{E}(r) = \exp\left(\mu_G + \frac{\sigma_G^2}{2}\right) = \frac{c}{24\eta} \;\Rightarrow\; \mu_G + \frac{\sigma_G^2}{2} = \log(\widetilde{i}) - \log(24\eta). \tag{35}$$

As a second restriction, we want our payment size distribution in the model to have the same coefficient of variation as the observed one. Let $\widehat{\mathbb{CV}}_r = \widehat{\sigma}_r / \widehat{\mathbb{E}}(r)$ denote the observed coefficient of variation of cash inflows. Our second restriction on parameters is then given by

$$\mathbb{CV}(r) = \frac{\sqrt{\mathbb{V}\mathrm{ar}(r)}}{\mathbb{E}(r)} = \sqrt{\left(\exp\left(\sigma_G^2\right) - 1\right)} = \widehat{\mathbb{CV}}_r, \tag{36}$$

which implies $\sigma_G^2 = \sqrt{\log\left(\widehat{\mathbb{CV}}_r^2 + 1\right)}$ and finally yields the calibrated scale and location parameters of G, i.e.,

$$\begin{aligned} \sigma_G &= \sqrt{\log\left(\widehat{\mathbb{CV}}_r^2 + 1\right)}, \\ \mu_G &= \log(\widetilde{i}) - \log(24\eta) - \frac{\log\left(\widehat{\mathbb{CV}}_r^2 + 1\right)}{2}. \end{aligned} \tag{37}$$

B.2 Identification of structural parameters

B.2.1 Non-identification of $\widetilde{\Theta}_c$

Proposition 2. *The set of parameters $\widetilde{\mathbf{\Theta}}_e = \{b, R, \kappa, \sigma_b, \sigma_\kappa, \psi\}$ is not identified.*

Proof. Let $\boldsymbol{\theta'} = \{b, R, \kappa, \sigma_b, \sigma_\kappa, \psi\}$ be a specific vector of parameters. Let $\boldsymbol{\theta''} = \left\{1, \frac{R}{b}, \frac{\kappa}{b}, \frac{\sigma_b}{b}, \frac{\sigma_\kappa}{b}, \psi\right\}$ be an alternative vector of parameters. Let $\boldsymbol{\pi}_{\boldsymbol{\theta'}} = \left(\pi_a^{\boldsymbol{\theta'}}(m), \pi_d^{\boldsymbol{\theta'}}(m,s)\right)$ be the choice probabilities associated with $\boldsymbol{\theta'}$, and similarly let $\boldsymbol{\pi}_{\boldsymbol{\theta''}} = \left(\pi_a^{\boldsymbol{\theta''}}(m), \pi_d^{\boldsymbol{\theta''}}(m,s)\right)$ be the choice probabilities associated with $\boldsymbol{\theta'}$. I want to show that $\boldsymbol{\pi}_{\boldsymbol{\theta'}} = \boldsymbol{\pi}_{\boldsymbol{\theta''}}$.

Let $\mathbf{v}_{\boldsymbol{\theta'}}$ and $\mathbf{v}_{\boldsymbol{\theta''}}$ be the set value functions associated with the two parameter vectors. First, we want to show that if $\mathbf{v}_{\boldsymbol{\theta''}} = \frac{\mathbf{v}_{\boldsymbol{\theta'}}}{b}$, then $\boldsymbol{\pi}_{\boldsymbol{\theta'}} = \boldsymbol{\pi}_{\boldsymbol{\theta''}}$. Adjustment probabilities are the same, since

$$\begin{aligned} \pi_a^{\boldsymbol{\theta'}}(m) =& \frac{\exp\left(\frac{v_n^{\boldsymbol{\theta'}}(m^*)-b}{\sigma_b}\right)}{\exp\left(\frac{v_n^{\boldsymbol{\theta'}}(m^*)-b}{\sigma_b}\right) + \exp\left(\frac{v_n^{\boldsymbol{\theta'}}(m)}{\sigma_b}\right)} = \frac{\exp\left(\frac{bv_n^{\boldsymbol{\theta''}}(m^*)-b}{\sigma_b}\right)}{\exp\left(\frac{bv_n^{\boldsymbol{\theta''}}(m^*)-b}{\sigma_b}\right) + \exp\left(\frac{bv_n^{\boldsymbol{\theta''}}(m)}{\sigma_b}\right)} = \\ =& \frac{\exp\left(\frac{v_n^{\boldsymbol{\theta''}}(m^*)-1}{\sigma_b/b}\right)}{\exp\left(\frac{v_n^{\boldsymbol{\theta''}}(m^*)-1}{\sigma_b/b}\right) + \exp\left(\frac{v_n^{\boldsymbol{\theta''}}(m)}{\sigma_b/b}\right)} = \pi_a^{\boldsymbol{\theta''}}(m) \end{aligned}$$

The same holds for cashless usage probabilities, as

$$
\begin{aligned}
\pi_d^{\boldsymbol{\theta}'}(m,s) =& \frac{\exp\left(\frac{\beta EV^{\boldsymbol{\theta}'}(m)-\kappa}{\sigma_\kappa}\right)}{\exp\left(\frac{\beta EV^{\boldsymbol{\theta}'}(m)-\kappa}{\sigma_\kappa}\right)+\exp\left(\frac{\beta EV^{\boldsymbol{\theta}'}(m-s)}{\sigma_\kappa}\right)} = \\
=& \frac{\exp\left(\frac{\beta bEV^{\boldsymbol{\theta}''}(m)-\kappa}{\sigma_\kappa}\right)}{\exp\left(\frac{\beta bEV^{\boldsymbol{\theta}''}(m)-\kappa}{\sigma_\kappa}\right)+\exp\left(\frac{\beta bEV^{\boldsymbol{\theta}''}(m-s)}{\sigma_\kappa}\right)} = \\
=& \frac{\exp\left(\frac{\beta EV^{\boldsymbol{\theta}''}(m)-\kappa/b}{\sigma_\kappa/b}\right)}{\exp\left(\frac{\beta EV^{\boldsymbol{\theta}''}(m)-\kappa/b}{\sigma_\kappa/b}\right)+\exp\left(\frac{\beta EV^{\boldsymbol{\theta}''}(m-s)}{\sigma_\kappa/b}\right)} = \pi_d^{\boldsymbol{\theta}''}(m,s)
\end{aligned}
$$

Finally, we actually show that $\mathbf{v}_{\boldsymbol{\theta}''} = \frac{\mathbf{v}_{\boldsymbol{\theta}'}}{b}$.We start from the function EV.

$$
\begin{aligned}
EV^{\boldsymbol{\theta}'}(m) =& \log\left(\exp\left(\frac{v_n^{\boldsymbol{\theta}'}(m^*)-b}{\sigma_b}\right)+\exp\left(\frac{v_n^{\boldsymbol{\theta}'}(m)}{\sigma_b}\right)\right)\sigma_b = \\
=& \log\left(\exp\left(\frac{bv_n^{\boldsymbol{\theta}''}(m^*)-b}{\sigma_b}\right)+\exp\left(\frac{bv_n^{\boldsymbol{\theta}''}(m)}{\sigma_b}\right)\right)\sigma_b = \\
=& \log\left(\exp\left(\frac{v_n^{\boldsymbol{\theta}''}(m^*)-1}{\sigma_b/b}\right)+\exp\left(\frac{v_n^{\boldsymbol{\theta}''}(m)}{\sigma_b/b}\right)\right)\sigma_b = bEV^{\boldsymbol{\theta}''}(m)
\end{aligned}
$$

Similarly, one can show that $U^{\boldsymbol{\theta}'}(m) = bU^{\boldsymbol{\theta}''}(m)$, and this can be used to show that $v_n^{\boldsymbol{\theta}'}(m) = bv_n^{\boldsymbol{\theta}''}$. This completes the proof. ■

B.2.2 Proof of Proposition 1

I now provide identification results for the set of structural parameters $\boldsymbol{\Theta}_e = \left\{\frac{R}{b}, \frac{\kappa}{b}, \frac{\sigma_b}{b}, \frac{\sigma_\kappa}{b}, \psi\right\}$, with a series of Lemmas that progressively show how these five parameters are uniquely pinned down by observables. We start by showing that $\frac{\sigma_b}{b}$ is identified by the log-odds of adjusting when cash balances are at the optimal level.

Lemma 1 (Identification of $\frac{\sigma_b}{b}$)**.** *The ratio $\frac{\sigma_b}{b}$ is identified from*

$$
\log\left(\frac{\pi_a(m^*)}{1-\pi_a(m^*)}\right) = -\frac{b}{\sigma_b}. \tag{38}
$$

Proof. Recall that the probability to adjust as a function of current cash holdings m is given by

$$\pi_a(m) = \frac{\exp(v_a/\sigma_b)}{\exp(v_a/\sigma_b) + \exp(v_n(m)/\sigma_b)}.$$

Therefore, we have that

$$\frac{\pi_a(m)}{1-\pi_a(m)} = \frac{\exp(v_a/\sigma_b)}{\exp(v_n(m)/\sigma_b)} \quad \Rightarrow \quad \log\left(\frac{\pi^a(m)}{1-\pi^a(m)}\right) = \frac{v_a - v_n(m)}{\sigma_b}.$$

Given that $v_a = v_n(m^*) - b$, this implies the desired result. Therefore, $\frac{\sigma_b}{b}$ is identified. ■

Let's briefly discuss the above result. The intuition is that, when $\frac{\sigma_b}{b}$ is very small, given that there is a positive fixed cost of adjusting, one should never see adjustments when the level of cash balances is the optimal one, and $\pi_a(m^*) \simeq 0$. When the unobserved components of adjustment decisions (e.g. mistakes, forced adjustments) are relevant enough, we might observe a positive probability of withdrawing when $m = m^*$. Notice, however, that no matter how large $\frac{\sigma_b}{b}$ is, we always have $\pi^a(m^*) < 1/2$.

We now move to the identification of $\frac{\sigma_\kappa}{\kappa}$, showing that this ratio is identified by the log-odds of paying with cards for tiny payments.

Lemma 2 (Idenfitication of $\frac{\sigma_\kappa}{\kappa}$)**.** *The ratio $\frac{\sigma_\kappa}{\kappa}$ is identified from*

$$\lim_{s\to 0^+} \log\left(\frac{\pi_d(m,s)}{1-\pi_d(m,s)}\right) = \frac{\kappa}{\sigma_\kappa}, \tag{39}$$

which holds for any $m > 0$.

Proof. Recall that probability of a cashless payment given current cash holdings m when facing a payment of size s is given by

$$\pi_d(m,s) = \frac{\exp((\beta EV(m) + \kappa)/\sigma_\kappa)}{\exp((\beta EV(m-s))/\sigma_\kappa) + \exp((\beta EV(m) - \kappa)/\sigma_\kappa)}.$$

Therefore, we have that

$$\frac{\pi_d(m,s)}{1-\pi_d(m,s)} = \frac{\exp((\beta EV(m) + \kappa)/\sigma_\kappa)}{\exp(\beta EV(m-s)/\sigma_\kappa)},$$

and therefore

$$\log\left(\frac{\pi_d(m,s)}{1-\pi_d(m,s)}\right) = \frac{\beta EV(m) + \kappa - \beta EV(m-s)}{\sigma_\kappa}$$
$$= \frac{\kappa}{\sigma_\kappa} + \frac{\beta}{\sigma_\kappa}\left(EV(m) - EV(m-s)\right),$$

Notice now that by taking the limit of this expression for $s \to 0$, we get that

$$\lim_{s\to 0^+} \log\left(\frac{\pi_d(m,s)}{1-\pi_d(m,s)}\right) = \frac{\kappa}{\sigma_\kappa}, \tag{40}$$

as desired. ■

The intuition for the above Lemma is that when s is very small the continuation value in case of a card or a cash payment is the same, so only the cost/benefits of using cards vs cash and the scale of payment choice shocks matter. Notice that Lemma 2 highlights a feature of the model: that the probability of using cards vs cash to settle tiny purchases is independent from the level of cash holdings m.

We now move to the identification of $\frac{\sigma_b}{\sigma_\kappa}$, showing that the relative magnitude of adjustment and payment shocks is identified by the log difference in card usage probabilities for (m, s) and (m, s'), rescaled by the discounted log difference in adjustment probabilities for $m - s$ and $m - s'$.

Lemma 3 (Identification of $\frac{\sigma_b}{\sigma_\kappa}$)**.** *The ratio $\frac{\sigma_b}{\sigma_\kappa}$ is identified by*

$$\frac{\log\left(\frac{\pi_d(m,s)}{1-\pi_d(m,s)}\right) - \log\left(\frac{\pi_d(m,s')}{1-\pi_d(m,s')}\right)}{\beta\log\left(\frac{\pi_a(m-s)}{\pi_a(m-s')}\right)} = \frac{\sigma_b}{\sigma_\kappa}, \tag{41}$$

for any m and for any $s \neq s'$.

Proof. Recall that

$$\pi_d(m,s) = \frac{\exp\left(\frac{\beta EV(m)-\kappa}{\sigma_\kappa}\right)}{\exp\left(\frac{\beta EV(m)-\kappa}{\sigma_\kappa}\right) + \exp\left(\frac{\beta EV(m-s)}{\sigma_\kappa}\right)}.$$

Hence, we have that

$$\log\left(\frac{\pi^d(m,s)}{1-\pi^d(m,s)}\right) - \log\left(\frac{\pi^d(m',s')}{1-\pi^d(m',s')}\right) = \frac{\beta}{\sigma_\kappa}\left(EV(m) - EV(m-s) - EV(m') + EV(m'-s')\right)$$

and if $m = m'$,

$$\log\left(\frac{\pi^d(m,s)}{1-\pi^d(m,s)}\right) - \log\left(\frac{\pi^d(m,s')}{1-\pi^d(m,s')}\right) = \frac{\beta}{\sigma_\kappa}\left(EV(m-s') - EV(m-s)\right)$$
$$= \frac{\beta\sigma_b}{\sigma_\kappa}\log\left(\frac{\exp\left(v_a/\sigma_b\right)+\exp\left(v_n(m-s')/\sigma_b\right)}{\exp\left(v_a/\sigma_b\right)+\exp\left(v_n(m-s)/\sigma_b\right)}\right),$$

where the last equality follows from the fact that

$$EV(m-s') - EV(m-s) = \sigma_b\left[\log\left(\exp\left(\frac{v_a}{\sigma_b}\right)+\exp\left(\frac{v_n(m'-s)}{\sigma_b}\right)\right) - \log\left(\exp\left(\frac{v_a}{\sigma_b}\right)+\exp\left(\frac{v_n(m-s)}{\sigma_b}\right)\right)\right].$$

Given that

$$\pi^a(m-s) = \frac{\exp(v_a/\sigma_b)}{\exp(v_a/\sigma_b)+\exp(v_n(m-s)/\sigma_b)}, \pi^a(m-s') = \frac{\exp(v_a/\sigma_b)}{\exp(v_a/\sigma_b)+\exp(v_n(m-s')/\sigma_b)},$$

we have that

$$\frac{\pi^a(m-s)}{\pi^a(m-s')} = \frac{\exp\left(v_a/\sigma_b\right)+\exp\left(v_n(m-s')/\sigma_b\right)}{\exp\left(v_a/\sigma_b\right)+\exp\left(v_n(m-s)/\sigma_b\right)}.$$

We therefore conclude that,

$$\log\left(\frac{\pi^d(m,s)}{1-\pi^d(m,s)}\right) - \log\left(\frac{\pi^d(m,s')}{1-\pi^d(m,s')}\right) = \frac{\beta\sigma_b}{\sigma_\kappa}\log\left(\frac{\pi^a(m-s)}{\pi^a(m-s')}\right),$$

which after manipulation yields the desired result. ■

I now provide an intuitive explanation for Lemma 3. First, notice that given that as the RHS must be positive (it is the ratio between two positive scale parameters), $\log\left(\frac{\pi^d(m,s)}{1-\pi^d(m,s)}\right) - \log\left(\frac{\pi^d(m,s')}{1-\pi^d(m,s')}\right)$ and $\log\left(\frac{\pi^a(m-s)}{\pi^a(m-s')}\right)$ must share the same sign. In particular, when the former is positive (it is more likely to pay by card for (m,s) than for (m,s')), then the latter must also be positive (it is more likely to adjust for $m-s$ than for $m-s'$). The intuition is that individuals prefer to settle with cards transactions that would increase adjustment probabilities in the future by a larger amount. When $\sigma_b \to +\infty$, the probability to adjust is independent from the amount of cash balances, and the denominator on the LHS approaches zero. When $\sigma_\kappa \to +\infty$, the probability to pay with cards is independent from cash holdings and purchase sizes, and the numerator on the LHS approaches zero. When the scale of shocks is identical ($\sigma_b = \sigma_\kappa$), we get that $\log\left(\frac{\pi^d(m,s)}{1-\pi^d(m,s)}\right) - \log\left(\frac{\pi^d(m,s')}{1-\pi^d(m,s')}\right) = \beta\log\left(\frac{\pi^a(m-s)}{\pi^a(m-s')}\right)$, i.e., that the log-odds difference in card usage probabilities for (m, s) and (m, s') equals the log difference in future adjustment probabilities, rescaled by β to account for for discounting. When the gap in card usage probability is larger than the discounted difference in future adjustment probabilities, this means that adjustment shocks are larger than payment choice shocks, and vice versa.

Notice that at this stage, we already know $\frac{\sigma_b}{b}$ from (38), $\frac{\kappa}{\sigma_\kappa}$ from (39) and $\frac{\sigma_b}{\sigma_\kappa}$ from (41). Hence, we easily get $\frac{\kappa}{b}$ via $\frac{\kappa}{b} = \frac{\kappa}{\sigma_\kappa} \cdot \frac{\sigma_\kappa}{\sigma_b} \cdot \frac{\sigma_b}{b}$, and $\frac{\sigma_\kappa}{b} = \frac{\sigma_\kappa}{\sigma_b} \cdot \frac{\sigma_b}{b}$. We are left with $\frac{R}{b}$ and ψ to identify. The identification of these last two parameters is discussed in the next, final Lemma.

Lemma 4. *The ratio $\frac{R}{\sigma_b}$ and ψ are identified by the following relationship. For any $m \neq m'$,*

$$\log\left(\frac{\pi^a(m)}{1-\pi^a(m)}\right) - \log\left(\frac{\pi^a(m')}{1-\pi^a(m')}\right) = \frac{R}{\sigma_b}(m-m') + \frac{\lambda\phi_c\psi b}{\sigma_b}\left(F(m') - F(m)\right) + \chi\left(\mathbf{\Theta}_c, \boldsymbol{\pi}, \frac{\sigma_\kappa}{b}, \frac{\sigma_b}{b}\right), \tag{42}$$

where χ is an object described in the proof, independent of $\frac{R}{\sigma_b}$ and ψ.

Proof. Recall that

$$\pi^a(m) = \frac{\exp(v_a/\sigma_b)}{\exp(v_a/\sigma_b) + \exp(v_n(m)/\sigma_b)}.$$

Therefore,

$$\frac{\pi^a(m)}{1-\pi^a(m)} = \frac{\exp(v_a/\sigma_b)}{\exp(v_n(m)/\sigma_b)} \Rightarrow \log\left(\frac{\pi^a(m)}{1-\pi^a(m)}\right) = \frac{v_a - v_n(m)}{\sigma_b}.$$

Hence, we have

$$\log\left(\frac{\pi^a(m)}{1-\pi^a(m)}\right) - \log\left(\frac{\pi^a(m')}{1-\pi^a(m')}\right) = \frac{v_n(m') - v_n(m)}{\sigma_b}.$$

We have now to rewrite $v_n(m') - v_n(m)$.

$$\begin{aligned}
v_n(m') - v_n(m) = & \quad -R(m'-m) + \lambda\left(EW(m') - EW(m)\right) + \\
& + \eta\beta\left(\int EV(m'+r)dG(s) - \int EV(m+r)dG(r)\right) + \\
& + (1-\lambda-\eta)\beta\left(EV(m') - EV(m)\right) = \\
= & \quad -R(m'-m) + \lambda\left(EW(m') - EW(m)\right) + \\
& + \eta\beta\int\left(EV(m'+r) - EV(m+r)\right)dG(r) + \\
& + (1-\lambda-\eta)\beta\sigma_b\log\left(\frac{\pi^a(m)}{\pi^a(m')}\right) = \\
= & \quad -R(m'-m) + \lambda\left(EW(m') - EW(m)\right) +
\end{aligned}$$

$$+\eta\beta\sigma_b \int \log\left(\frac{\pi^a(m+r)}{\pi^a(m'+r)}\right) dG(r)+$$
$$+(1-\lambda-\eta)\beta\sigma_b \log\left(\frac{\pi^a(m)}{\pi^a(m')}\right).$$

We now focus on $EW(m) - EW(m')$, assuming WLG that $m > m'$.

$$\begin{aligned}
EW(m) - EW(m') = \quad & \phi_d\beta\left(EV(m) - EV(m')\right) + \\
& + \phi_c\left((1-F(m)) - \left(1-F(m')\right)\right)\left(\beta EV(m^*) - \psi b\right) + \\
& + \phi_c\beta\left(\int_0^m EV(m-s)dF(s) - \int_0^{m'} EV(m'-s)dF(s)\right) + \\
& + \phi_{cd}\left((1-F(m))\left(\beta EV(m)+\kappa\right) - \left(1-F(m')\right)\left(\beta EV(m')+\kappa\right)\right) + \\
& + \phi_{cd}\left(\int_0^m U(m,s)dF(s) - \int_0^{m'} U(m',s)dF(s)\right) = \\
= \quad & \phi_d\beta\sigma_b\log\left(\frac{\pi^a(m')}{\pi^a(m)}\right) + \\
& + \phi_c\left(F(m') - F(m)\right)\left(\beta EV(m^*) - \psi b\right) + \\
& + \phi_c\beta\left(\int_0^{m'}\left(EV(m-s) - EV(m'-s)\right)dF(s) + \int_{m'}^m EV(m-s)dF(s)\right) + \\
& + \phi_{cd}\left(1-F(m)\right)\beta\left(EV(m) - EV(m')\right) + \\
& + \phi_{cd}\left(F(m') - F(m)\right)\left(\beta EV(m')+\kappa\right) + \\
& + \phi_{cd}\left(\int_0^{m'}\left(U(m,s) - U(m',s)\right)dF(s) + \int_{m'}^m U(m,s)dF(s)\right) = \\
= \quad & \phi_d\beta\sigma_b\log\left(\frac{\pi^a(m')}{\pi^a(m)}\right) + \\
& + \phi_c\left(F(m) - F(m')\right)\psi + \\
& + \phi_c\beta\sigma_b\int_0^{m'}\log\left(\frac{\pi^a(m'-s)}{\pi^a(m-s)}\right)dF(s) + \\
& + \phi_c\beta\int_{m'}^m\left(EV(m-s) - EV(m^*)\right)dF(s) + \\
& + \phi_{cd}\beta\sigma_b\left(1-F(m)\right)\log\left(\frac{\pi^a(m')}{\pi^a(m)}\right) + \\
& + \phi_{cd}\left(F(m') - F(m)\right)\left(\beta EV(m')+\kappa\right) + \\
& + \phi_{cd}\left(\int_0^{m'}\left(U(m,s) - U(m',s)\right)dF(s) + \int_{m'}^m U(m,s)dF(s)\right)
\end{aligned}$$

Notice now that

$$U(m,s)-U(m',s)=\sigma_\kappa \log\left(\frac{\exp\left(\frac{v_c(m,s)}{\sigma}\right)+\exp\left(\frac{v_d(m)}{\sigma}\right)}{\exp\left(\frac{v_c(m',s)}{\sigma}\right)+\exp\left(\frac{v_d(m')}{\sigma}\right)}\right)= \\ =\sigma_\kappa \log\left(\frac{\pi^d(m',s)}{\pi^d(m,s)}\right)$$

We therefore obtain

$$\begin{aligned} EW(m)-EW(m') = \quad & \phi_d\beta\sigma_b \log\left(\frac{\pi^a(m')}{\pi^a(m)}\right)+\phi_c\left(F(m)-F(m')\right)\psi b+ \\ & +\phi_c\beta\sigma_b\int_0^{m'}\log\left(\frac{\pi^a(m'-s)}{\pi^a(m-s)}\right)dF(s)+ \\ & +\phi_c\beta\sigma_b\int_{m'}^{m}\log\left(\frac{\pi^a(m^*)}{\pi^a(m-s)}\right)dF(s)+ \\ & +\phi_{cd}\beta\sigma_b\left(1-F(m)\right)\log\left(\frac{\pi^a(m')}{\pi^a(m)}\right)+ \\ & +\phi_{cd}\sigma_\kappa\int_0^{m'}\log\left(\frac{\pi^d(m',s)}{\pi^d(m,s)}\right)dF(s)+ \\ & +\phi_{cd}\int_{m'}^{m}\left(U(m,s)-\left(\beta EV(m')+\kappa\right)\right)dF(s) \end{aligned}$$

Finally, notice that since

$$\beta EV(m')+\kappa=\sigma_\kappa\log\left(\exp\left(\frac{\beta EV(m')+\kappa}{\sigma_\kappa}\right)\right),$$

we have that

$$\begin{aligned} U(m,s)-\beta EV(m')-\kappa &= \sigma_\kappa\log\left(\exp\left(\frac{\beta EV(m-s)}{\sigma_\kappa}\right)+\exp\left(\frac{\beta EV(m)+\kappa}{\sigma_\kappa}\right)\right)-\beta EV(m')-\kappa= \\ &= \sigma_\kappa\log\left(\frac{\exp\left(\frac{\beta EV(m-s)}{\sigma_\kappa}\right)+\exp\left(\frac{\beta EV(m)+\kappa}{\sigma_\kappa}\right)}{\exp\left(\frac{\beta EV(m')+\kappa}{\sigma_\kappa}\right)}\right)= \\ &= \sigma_\kappa\log\left(\frac{\exp\left(\frac{\beta EV(m)+\kappa}{\sigma_\kappa}\right)}{\pi^d(m,s)\exp\left(\frac{\beta EV(m')+\kappa}{\sigma_\kappa}\right)}\right)= \\ &= \sigma_\kappa\left[\log\left(\frac{\exp\left(\frac{\beta EV(m)+\kappa}{\sigma_\kappa}\right)}{\exp\left(\frac{\beta EV(m')+\kappa}{\sigma_\kappa}\right)}\right)-\log\left(\pi^d(m,s)\right)\right]= \\ &= \sigma_\kappa\left[\frac{\beta}{\sigma_\kappa}\left(EV(m)-EV(m')\right)-\log\left(\pi^d(m,s)\right)\right]= \\ &= \beta\sigma_b\log\left(\frac{\pi^a(m')}{\pi^a(m)}\right)-\sigma_\kappa\log\left(\pi^d(m,s)\right). \end{aligned}$$

Thus,

$$\begin{aligned}EW(m)-EW(m') = \quad & \phi_d\beta\sigma_b \log\left(\frac{\pi^a(m')}{\pi^a(m)}\right)+\phi_c\left(F(m)-F(m')\right)\psi b+\\
&+\phi_c\beta\sigma_b\int_0^{m'}\log\left(\frac{\pi^a(m'-s)}{\pi^a(m-s)}\right)dF(s)+\\
&+\phi_c\beta\sigma_b\int_{m'}^{m}\log\left(\frac{\pi^a(m^*)}{\pi^a(m-s)}\right)dF(s)+\\
&+\phi_{cd}\beta\sigma_b\left(1-F(m)\right)\log\left(\frac{\pi^a(m')}{\pi^a(m)}\right)+\\
&+\phi_{cd}\sigma_\kappa\int_0^{m'}\log\left(\frac{\pi^d(m',s)}{\pi^d(m,s)}\right)dF(s)+\\
&+\phi_{cd}\int_{m'}^{m}\left(\beta\sigma_b\log\left(\frac{\pi^a(m')}{\pi^a(m)}\right)-\sigma_\kappa\log\left(\pi^d(m,s)\right)\right)dF(s)\end{aligned}$$

and simplifying

$$\begin{aligned}EW(m)-EW(m') = \quad & \phi_d\beta\sigma_b \log\left(\frac{\pi^a(m')}{\pi^a(m)}\right)+\phi_c\left(F(m)-F(m')\right)\psi b+\\
&+\phi_c\beta\sigma_b\int_0^{m'}\log\left(\frac{\pi^a(m'-s)}{\pi^a(m-s)}\right)dF(s)+\\
&+\phi_c\beta\sigma_b\int_{m'}^{m}\log\left(\frac{\pi^a(m^*)}{\pi^a(m-s)}\right)dF(s)+\\
&+\phi_{cd}\beta\sigma_b\left(1-F(m)\right)\log\left(\frac{\pi^a(m')}{\pi^a(m)}\right)+\\
&+\phi_{cd}\sigma_\kappa\int_0^{m'}\log\left(\frac{\pi^d(m',s)}{\pi^d(m,s)}\right)dF(s)+\\
&+\phi_{cd}\beta\sigma_b\left(F(m)-F(m')\right)\log\left(\frac{\pi^a(m')}{\pi^a(m)}\right)+\\
&-\phi_{cd}\sigma_\kappa\int_{m'}^{m}\log\left(\pi^d(m,s)\right)dF(s).\end{aligned}$$

We can finally rewrite

$$\begin{aligned}v_n(m')-v_n(m) = & -R(m'-m)+\\
&+\lambda\phi_d\beta\sigma_b\log\left(\frac{\pi^a(m)}{\pi^a(m')}\right)+\\
&+\lambda\phi_c\left(F(m')-F(m)\right)\psi b+\\
&+\lambda\phi_c\beta\sigma_b\int_0^{m'}\log\left(\frac{\pi^a(m-s)}{\pi^a(m'-s)}\right)dF(s)+\\
&+\lambda\phi_c\beta\sigma_b\int_{m'}^{m}\log\left(\frac{\pi^a(m-s)}{\pi^a(m^*)}\right)dF(s)+\end{aligned}$$

$$
\begin{aligned}
&+\lambda\phi_{cd}\beta\sigma_b\left(1-F(m)\right)\log\left(\frac{\pi^a(m)}{\pi^a(m')}\right)+\\
&+\lambda\phi_{cd}\sigma_\kappa\int_0^{m'}\log\left(\frac{\pi^d(m,s)}{\pi^d(m',s)}\right)dF(s)+\\
&+\lambda\phi_{cd}\beta\sigma_b\left(F(m)-F(m')\right)\log\left(\frac{\pi^a(m)}{\pi^a(m')}\right)+\\
&+\lambda\phi_{cd}\sigma_\kappa\int_{m'}^{m}\log\left(\pi^d(m,s)\right)dF(s)+\\
&+\eta\beta\sigma_b\int\log\left(\frac{\pi^a(m+s)}{\pi^a(m'+s)}\right)dF(s)+\\
&+(1-\lambda-\eta)\beta\sigma_b\log\left(\frac{\pi^a(m)}{\pi^a(m')}\right).
\end{aligned}
$$

We obtain

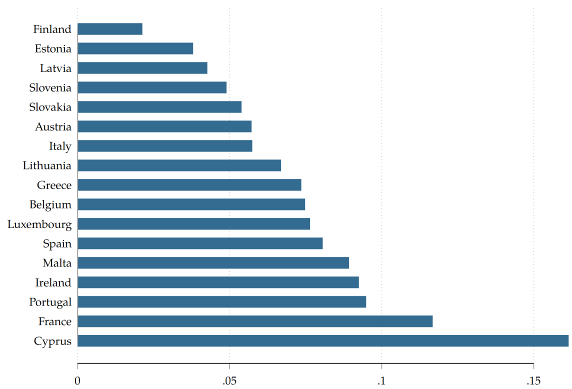
$$
\log\left(\frac{\pi^a(m)}{1-\pi^a(m)}\right)-\log\left(\frac{\pi^a(m')}{1-\pi^a(m')}\right)=\frac{R}{\sigma_b}(m-m')+\frac{\lambda\phi_c\psi b}{\sigma_b}\left(F(m')-F(m)\right)+\chi\left(\boldsymbol{\Theta}_c,\boldsymbol{\pi},\frac{\sigma_\kappa}{b},\frac{\sigma_b}{b}\right), \tag{43}
$$

where $\boldsymbol{\pi}$ is the set of choice probabilities and $\chi\left(\boldsymbol{\Theta}_c,\boldsymbol{\pi},\frac{\sigma_\kappa}{b},\frac{\sigma_b}{b}\right)$ collects all terms which do not depend on $\frac{R}{\sigma_b}$ and ψ. Notice that since all terms in $\chi\left(\boldsymbol{\Theta}_c,\boldsymbol{\pi},\frac{\sigma_\kappa}{b},\frac{\sigma_b}{b}\right)$ are already known at this stage (some from the data, some from previous steps of the identification procedure), this constitute a set of equations (one for each pair of m and m') which enable me to identify the two remaining parameters. ■

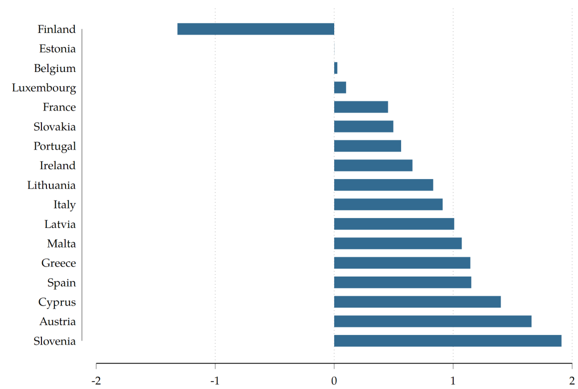
C Quantitative model: additional figures

Figure C.1: Estimated parameters $\widehat{\Theta}_e$ by country.

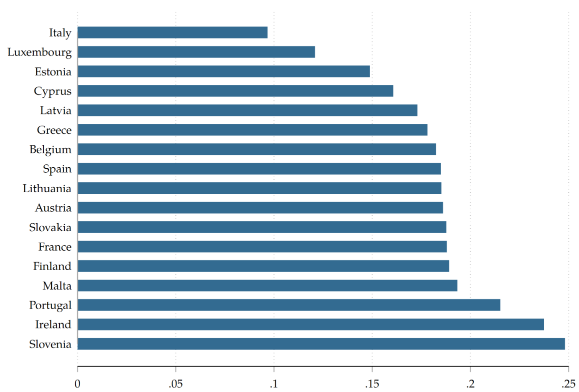
(a) Yearly opportunity cost R



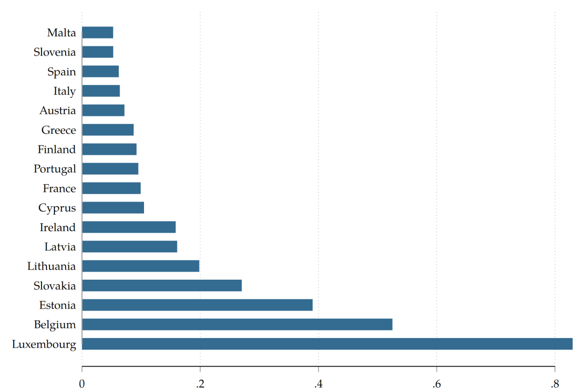
(b) Card usage cost/benefit κ/b



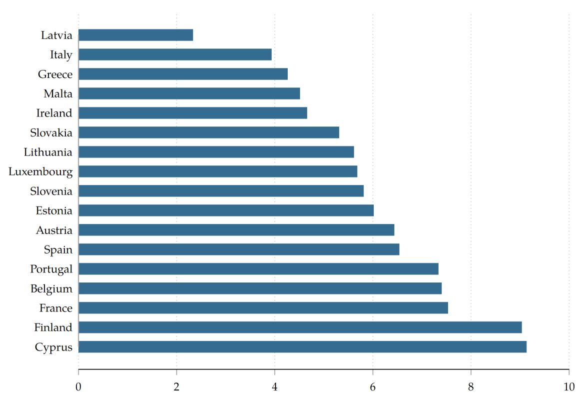
(c) Adjustment shock scale $\mathbb{SD}(\epsilon)/b$



(d) Payment shock scale $\mathbb{SD}(\chi)/|\kappa|$

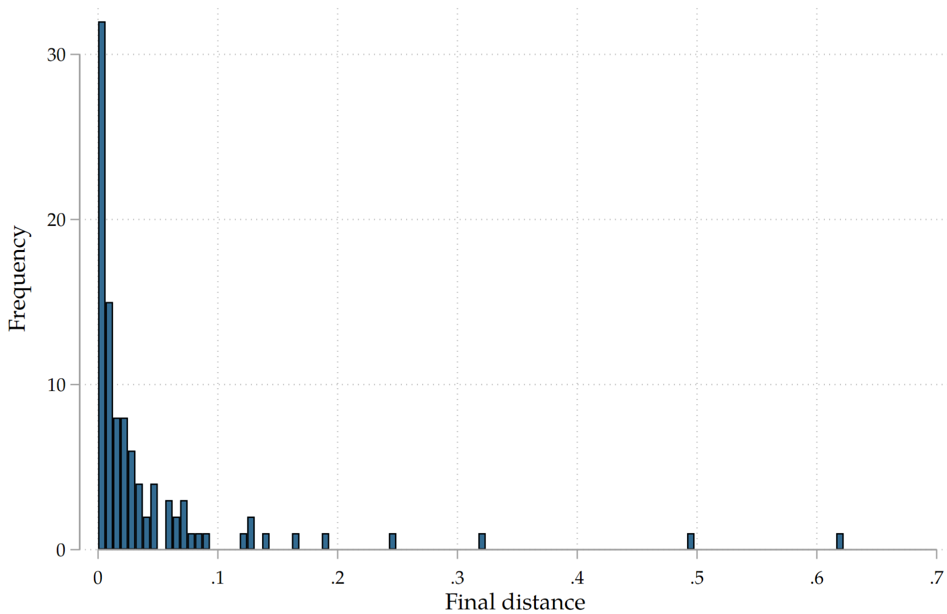


(e) Withdrawal penalty ψ



Note: This Figure complements Figure 9 and Figure 10 by displaying the country-level distribution of my estimates for the five model parameters. Country-level parameter estimates are obtained through a population-weighted mean of region level parameter estimates.

FIGURE C.2: Minimum distance at parameter estimates.



Note: This Figure displays an histogram of distances between model-implied and empirical moments at the estimated parameter vector, for each region. It is given by the sum of squared percentage deviations for each of the five moments. The median distance is .013, so the median squared percentage deviation is .0026, corresponding to a percentage deviation of around 5%.

FIGURE C.3: Estimated parameters $\widehat{\Theta}_e$ - variation between and within countries.